ROBUST EIGENVECTOR ALGORITHMS FOR BLIND DECONVOLUTION OF MIMO LINEAR CHANNELS

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ABSTRACT

This paper presents eigenvector algorithms (EVAs) for blind deconvolution of multiple-input multiple-output infinite impulse response (MIMO-IIR) channels (convolutive mixtures). One of the attractive features of the proposed EVA is that it is insensitive to Gaussian noises which are added to the outputs of the channels; hence the proposed EVA is referred to as a "robust" eigenvector algorithm (REVA). Simulation results show the validity of the REVA.

Index Terms— Eigenvector algorithms, Robust eigenvector algorithms, Blind deconvolution, MIMO-IIR channels, Gaussian noise

1. INTRODUCTION

In this paper, we deal with a blind deconvolution (BD) problem for multiple-input and multiple-output (MIMO) infinite-impulse response (IIR) channels. To solve this problem, we use eigenvector algorithms (EVAs) [4, 7]. The first proposal of the EVA was done by Jelonnek et al. [4]. They have proposed the EVA for solving blind equalization (BE) problems of single-input single-output (SISO) channels or single-input multiple-output (SIMO) channels. In [7], several procedures for the blind source separation (BSS) of instantaneous mixtures, using the generalized eigenvalue decomposition, have been introduced. Recently, the authors have proposed an EVA which can solve BSS problems in the case of MIMO static systems (instantaneous mixtures) [5, 6].

In this paper, based on the idea in [5, 6], we shall show that EVAs can be used to solve the BD problem of MIMO-IIR channels. Moreover, it will be shown that the proposed EVA has such an attractive feature that the BD can be achieved with as little influence of Gaussian noise as possible; hence this type of EVA is referred to as a "robust" EVA (REVA). Compute simulations are presented to demonstrate the validity of the REVA.

The present paper uses the following notation: Let Z denote the set of all integers. Let C denote the set of all complex numbers. Let $\boldsymbol{C}^n$ denote the set of all n-column vectors with complex components. Let $\boldsymbol{C}^{m\times n}$ denote the set of all $m \times n$ matrices with complex components. The superscripts T, $*$, and H denote, respectively, the transpose, the complex conjugate, and the complex conjugate transpose (Hermitian) of a matrix. The symbols block-diag$\{\cdots\}$ and diag$\{\cdots\}$ denote respectively a block diagonal and a diagonal matrices with the block diagonal and the diagonal elements $\{\cdots\}$. The symbol cum$\{x_1,x_2,x_3,x_4\}$ denotes a fourth-order cumulant of x_i's. Let $i = \overline{1,n}$ stands for $i = 1, 2, \cdots, n$.

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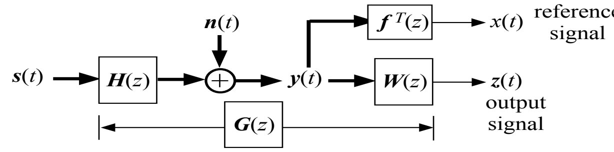

Fig. 1. The composite system of an unknown system and a deconvolver, and a reference system.

2. PROBLEM FORMULATION AND ASSUMPTIONS

We consider an MIMO channel with n inputs and m outputs as described by

$$\boldsymbol{y}(t) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k) + \boldsymbol{n}(t), \quad t \in Z, \tag{1}$$

where $\boldsymbol{s}(t)$ is an n-column vector of input (or source) signals, $\boldsymbol{y}(t)$ is an m-column vector of channel outputs, $\boldsymbol{n}(t)$ is an m-column vector of Gaussian noises, and $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ impulse response matrix sequence. The transfer function of the channel is defined by $\boldsymbol{H}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} z^k, z \in C$.

To recover the source signals, we process the output signals by an $n \times m$ deconvolver (or equalizer) $\boldsymbol{W}(z)$ described by

$$\begin{aligned}\boldsymbol{z}(t) &= \sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{y}(t-k) \\ &= \sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} \boldsymbol{s}(t-k) + \sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{n}(t-k),\end{aligned} \tag{2}$$

where $\{\boldsymbol{G}^{(k)}\}$ is the impulse response matrix sequence of $\boldsymbol{G}(z) := \boldsymbol{W}(z)\boldsymbol{H}(z)$, which is defined by $\boldsymbol{G}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} z^k, z \in C$. The cascade connection of the unknown system and the deconvolver is illustrated in Fig. 1.

Here, we put the following assumptions on the channel, the source signals, the deconvolver, and the noises.

A1) The transfer function $\boldsymbol{H}(z)$ is stable and has full column rank on the unit circle $|z| = 1$, where the assumption **A1)** implies that the unknown system has less inputs than outputs, i.e., $n \le m$, and there exists a left stable inverse of the unknown system.

A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a complex, zero-mean and non-Gaussian random vector process with element processes

$\{s_i(t)\}$, $i=\overline{1,n}$ being mutually independent. Each element process $\{s_i(t)\}$ is an i.i.d. process with a variance $\sigma^2_{s_i}\neq 0$ and a nonzero fourth-order cumulant $\gamma_i\neq 0$ defined as

$$\gamma_i = \text{cum}\{s_i(t), s_i(t), s_i^*(t), s_i^*(t)\} \neq 0. \quad (3)$$

A3) The deconvolver $\boldsymbol{W}(z)$ is an FIR channel of sufficient length L so that the truncation effect can be ignored.
A4) The noise sequence $\{\boldsymbol{n}(t)\}$ is a zero-mean, Gaussian vector stationary process whose component processes $\{n_j(t)\}$, $j=\overline{1,m}$ have nonzero variances $\sigma^2_{n_j}$, $j=\overline{1,m}$.
A5) The two vector sequences $\{\boldsymbol{n}(t)\}$ and $\{\boldsymbol{s}(t)\}$ are mutually statistically independent.

Under **A3)**, the impulse responses $\boldsymbol{G}^{(k)}$ for $k\in Z$ of the cascade system are given by

$$\boldsymbol{G}^{(k)} := \textstyle\sum_{\tau=L_1}^{L_2} \boldsymbol{W}^{(\tau)}\boldsymbol{H}^{(k-\tau)}, \quad k\in Z, \quad (4)$$

where the length $L := L_2 - L_1 + 1$ is taken to be sufficiently large. In a vector form, (4) can be written as

$$\tilde{\boldsymbol{g}}_i = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{w}}_i, \quad i=\overline{1,n}, \quad (5)$$

where $\tilde{\boldsymbol{g}}_i$ is the column vector consisting of the ith output impulse response of the cascade system defined by $\tilde{\boldsymbol{g}}_i := [\boldsymbol{g}_{i1}^T, \boldsymbol{g}_{i2}^T, \cdots, \boldsymbol{g}_{in}^T]^T$,

$$\boldsymbol{g}_{ij} := [\cdots, g_{ij}(-1), g_{ij}(0), g_{ij}(1), \cdots]^T, \quad j=\overline{1,n} \quad (6)$$

where $g_{ij}(k)$ is the (i,j)th element of matrix $\boldsymbol{G}^{(k)}$, and $\tilde{\boldsymbol{w}}_i$ is the mL-column vector consisting of the tap coefficients (corresponding to the ith output) of the deconvolver defined by $\tilde{\boldsymbol{w}}_i := \left[\boldsymbol{w}_{i1}^T, \boldsymbol{w}_{i2}^T, \cdots, \boldsymbol{w}_{im}^T\right]^T \in \boldsymbol{C}^{mL}$,

$$\boldsymbol{w}_{ij} := [w_{ij}(L_1), w_{ij}(L_1+1), \cdots, w_{ij}(L_2)]^T \in \boldsymbol{C}^L, \quad (7)$$

$j=\overline{1,m}$, where $w_{ij}(k)$ is the (i,j)th element of matrix $\boldsymbol{W}^{(k)}$, and $\tilde{\boldsymbol{H}}$ is the $n\times m$ block matrix whose (i,j)th block element $\boldsymbol{H}_{ij}$ is the matrix (of L columns and possibly infinite number of rows) with the (l,r)th element $[\boldsymbol{H}_{ij}]_{lr}$ defined by $[\boldsymbol{H}_{ij}]_{lr} := h_{ji}(l-r)$, $l=0,\pm1,\pm2,\cdots$, $r=\overline{L_1,L_2}$, where $h_{ij}(k)$ is the (i,j)th element of the matrix $\boldsymbol{H}^{(k)}$.

In the multichannel blind deconvolution problem, we want to adjust $\tilde{\boldsymbol{w}}_i$'s (i = $\overline{1,n}$) so that

$$[\tilde{\boldsymbol{g}}_1, \cdots, \tilde{\boldsymbol{g}}_n] = \tilde{\boldsymbol{H}}[\tilde{\boldsymbol{w}}_1, \cdots, \tilde{\boldsymbol{w}}_n] = [\tilde{\boldsymbol{\delta}}_1, \cdots, \tilde{\boldsymbol{\delta}}_n]\boldsymbol{P}, \quad (8)$$

where $\boldsymbol{P}$ is an $n\times n$ permutation matrix, and $\tilde{\boldsymbol{\delta}}_i$ is the n-block column vector defined by

$$\tilde{\boldsymbol{\delta}}_i := [\boldsymbol{\delta}_{i1}^T, \boldsymbol{\delta}_{i2}^T, \ldots, \boldsymbol{\delta}_{in}^T]^T, \quad i=\overline{1,n} \quad (9)$$

$$\boldsymbol{\delta}_{ij} := \begin{cases} \hat{\boldsymbol{\delta}}_i, & \text{if } i=j, \\ (\cdots,0,0,0,\cdots)^T, & \text{otherwise.} \end{cases} \quad (10)$$

Here, $\hat{\boldsymbol{\delta}}_i$ is the column vector (of infinite elements) whose rth element $\hat{\delta}_i(r)$ is given by $\hat{\delta}_i(r) = d_i\delta(r-k_i)$, where $\delta(t)$ is the Kronecker delta function, d_i is a complex number standing for a scale change and a phase shift, and k_i is an integer standing for a time shift.

3. EIGENVECTOR ALGORITHMS (REVAS)

3.1. Robust Eigenvector algorithm

In order to solve the BD problem, the following cross-cumulant between $z_i(t)$ and a reference signal $x(t)$ (see Fig. 1) is defined;

$$D_{zx} = \text{cum}\{z_i(t), z_i^*(t), x(t), x^*(t)\}, \quad (11)$$

where $z_i(t)$ is the ith element of $\boldsymbol{z}(t)$ in (2) and the reference signal $x(t)$ is given by $\boldsymbol{f}^T(z)\boldsymbol{y}(t)$, using an appropriate filter $\boldsymbol{f}(z)$. The filter $\boldsymbol{f}(z)$ is called a *reference system*. Let $\boldsymbol{a}(z) := \boldsymbol{H}^T(z)\boldsymbol{f}(z) = [a_1(z), a_2(z), \cdots, a_n(z)]^T$, then $x(t) = \boldsymbol{f}^T(z)\boldsymbol{H}(z)\boldsymbol{s}(t) = \boldsymbol{a}^T(z)\boldsymbol{s}(t)$. The element $a_j(z)$ of the filter $\boldsymbol{a}(z)$ is defined as $a_j(z) = \sum_{k=-\infty}^{\infty} a_j(k)z^k$ and the reference system $\boldsymbol{f}(z)$ is an m-column vector whose elements are $f_j(z) = \sum_{k=L_1}^{L_2} f_j(k)z^k$, $j=\overline{1,m}$.

Researches using the idea of reference signals to solve the blind signal processing (BSP) problem, to our best knowledge, have been made by Adib et al. (e.g., [1]), Rhioui [8], and Jelonnek et al. (e.g., [4]). Adib et al. have shown that the BSS for instantaneous mixtures can be achieved by maximizing $|D_{zx}|$ in (11) under the constraint $\sigma^2_{z_i} = \sigma^2_{s_{\rho_i}}$, but they have not proposed any algorithm for achieving this idea, where $\sigma^2_{z_i}$ and $\sigma^2_{s_{\rho_i}}$ denote the variances of the output $z_i(t)$ and a source signal $s_{\rho_i}(t)$, respectively, and ρ_i is one of integers $\{1,2,\cdots,n\}$ such that the set $\{\rho_1, \rho_2, \cdots, \rho_n\}$ is a permutation of the set $\{1, 2, \cdots, n\}$. Rhioui et al. have proposed quadratic MIMO contrast functions for the BSS with convolutive mixtures. In their method, the number of reference signals corresponds to the number of source signals which can be extracted. Moreover, they claimed that as the reference signal, it is a practical valid choice to choose a signal obtained by whitening the outputs of the MIMO convolved system. Jelonnek et al. have shown in the single-input case that by the Lagrangian method, the maximization of $|D_{zx}|$ under $\sigma^2_{z_i} = \sigma^2_{s_{\rho_i}}$ leads to a closed-form expressed as a generalized eigenvector problem, which is referred to as an eigenvector algorithm (EVA). The EVA can solve the BE problems for SISO or SIMO channels.

In this paper, under the assumption that any reference system $\boldsymbol{f}(z)$ is used, we want to show how the EVA works for the BD of the MIMO-IIR channel (1). Moreover, it is shown that the proposed EVA has such a property that it works as little sensitive to Gaussian noises as possible.

To this end, we introduce fourth-order cumulants matrices of m-vector random process $\{\boldsymbol{y}(t)\}$ [9], which constitute a set of $m\times m$ block matrices $\boldsymbol{F}^{(4)}_{\boldsymbol{y},j,l}$ whose elements are defined by

$$\left[\boldsymbol{F}^{(4)}_{\boldsymbol{y},j,l}\right]_{[p,q]_{l_1 l_2}}$$
$$= \text{cum}\{y_q(t\text{-}L_1\text{-}l_2+1), y_p^*(t\text{-}L_1\text{-}l_1+1), y_j(t\text{-}l), y_j^*(t\text{-}l)\},$$
$$p,q,j=\overline{1,m}, \quad l_1,l_2=\overline{1,L}, \quad l=\overline{L_1,L_2}, \quad (12)$$

where $[\cdot]_{[p,q]_{l_1 l_2}}$ denotes the (l_1,l_2)th element of the (p,q)th block matrix of the matrix $\boldsymbol{F}^{(4)}_{\boldsymbol{y},j,l}$. Then, we consider an $m\times m$ block matrix $\tilde{\boldsymbol{F}}$ expressed by $\tilde{\boldsymbol{F}} = \sum_{j=1}^{m}\sum_{l=L_1}^{L_2} \boldsymbol{F}^{(4)}_{\boldsymbol{y},j,l}$. It is shown by a simple calculation that $\tilde{\boldsymbol{F}}$ becomes

$$\tilde{\boldsymbol{F}} = \tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{\Psi}} \tilde{\boldsymbol{H}}, \tag{13}$$

where $\tilde{\boldsymbol{\Psi}}$ is the diagonal matrix defined by

$$\tilde{\boldsymbol{\Psi}} := \text{block-diag}\{\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \cdots, \boldsymbol{\Psi}_n\}, \tag{14}$$

$$\boldsymbol{\Psi}_i := \text{diag}\{\cdots, \gamma_i\tilde{a}_i(\text{-}1), \gamma_i\tilde{a}_i(0), \gamma_i\tilde{a}_i(1), \cdots\}, i = \overline{1,n}, \tag{15}$$

$$\tilde{a}_i(k) := \textstyle\sum_{j=1}^{m}\sum_{l=L_1}^{L_2}|h_{ji}(k-l)|^2, i = \overline{1,n}, k \in Z. \tag{16}$$

It should be noted from the assumption **A1)** that all $\tilde{a}_i(k)$ are positive, if $L_1 = -\infty$ and $L_2 = \infty$.

Here, as a constraint, we take the following value;

$$|D_{zy}| = |\textstyle\sum_{j=1}^{m}\sum_{l=L_1}^{L_2}\text{cum}\{z_i(t), z_i^*(t), y_j(t\text{-}l), y_j^*(t\text{-}l)\}|$$
$$= |\boldsymbol{w}_i^T\tilde{\boldsymbol{F}}\boldsymbol{w}_i| = |\textstyle\sum_{r=1}^{n}\gamma_r\sum_{k=-\infty}^{\infty}\tilde{a}_r(k)|g_{ir}(k)|^2|. \tag{17}$$

Then, we consider of solving the problem that the fourth-order cumulant $|D_{zx}|$, which can be decomposed as $D_{zx} = \tilde{\boldsymbol{w}}_i^H\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_i$, is maximized under the condition, that is, $|D_{zy}| = |\gamma_{\rho_i}\tilde{a}_{\rho_i}(k)|$. It should be noted that we may choose an appropriate positive value for $\tilde{a}_{\rho_i}(k)$, if its true value is not available. By the Lagrangian method, the following generalized eigenvector problem is derived from the above problem;

$$\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_i = \lambda_i\tilde{\boldsymbol{F}}\tilde{\boldsymbol{w}}_i. \tag{18}$$

where $\tilde{\boldsymbol{B}}$ is the $m \times m$ block matrix whose (i,j)th block element $\boldsymbol{B}_{ij}$ is the matrix with the (l,r)th element $[\boldsymbol{B}_{ij}]_{lr}$ calculated by $\text{cum}\{y_i^*(t-L_1-l+1), y_j(t-L_1-r+1), x^*(t), x(t)\}$ $(l, r = \overline{1,L})$.

From the following theorem, one can see that by solving the eigenvector problem of the matrix $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$, its solution provides the vectors $\tilde{\boldsymbol{w}}_i$ $(i = \overline{1,n})$ satisfying (8). In the following theorem, we confine ourselves to the case $m = n$ for the simplicity of discussion, although our results are expandable to the case $m > n$. Let the eigenvalues of the diagonal matrix $\tilde{\boldsymbol{\Psi}}^{-1}\tilde{\boldsymbol{\Lambda}}$ be denoted by

$$\mu_i(k) := |a_i(k)|^2/\tilde{a}_i(k), \qquad i = \overline{1,n}, \quad k \in Z. \tag{19}$$

Theorem 1 *Assume $L_1 = -\infty$ and $L_2 = \infty$, and suppose the following conditions holds true:*

T1) *All the eigenvalues $\mu_i(k)$'s are distinct for $i = \overline{1,n}$ and $k \in Z$.*

Then the n eigenvectors corresponding to n nonzero eigenvalues $\mu_i(k_i)$'s of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$ become the vectors $\tilde{\boldsymbol{w}}_i$, $i = \overline{1,n}$, satisfying (8).

The proof of Theorem 1 is omitted for the page limit.

Remark 1 *Since the matrix $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$ consists of only fourth-order cumulants, the eigenvectors derived from the matrix can be obtained with as little influence of Gaussian noise as possible, which is referred as a robust eigenvector algorithm (REVA).*

Remark 2 *The proposed EVA is closely related to the joint diagonalization of square matrices (e.g., [2]).*

Remark 3 *It can be easily proved from Theorem 1that if the assumption* **T1)** *holds true, among all eigenvectors of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$, there exist the eigenvectors that can extract the same source signal from the output $\boldsymbol{y}(t)$. Therefore, in the next subsection, we shall show how to choose the eigenvectors corresponding to $\tilde{\boldsymbol{w}}_i$, $i = \overline{1,n}$, satisfying (8) from all eigenvectors of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$.*

3.2. How to choose the eigenvectors

In the previous subsection, in order to obtain the deconvolver $\tilde{\boldsymbol{w}}_i$ in (8), it was shown that the eigenvectors of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$ were calculated. In this subsection, in order to show how to choose such eigenvectors that the solutions (8) can be obtained, we consider the following eigenvector problem;

$$\tilde{\boldsymbol{B}}\tilde{\boldsymbol{F}}^{-1}\hat{\boldsymbol{w}}_i = \hat{\lambda}_i\hat{\boldsymbol{w}}_i, \tag{20}$$

where the structure of $\hat{\boldsymbol{w}}_i$ is the same as the one of $\tilde{\boldsymbol{w}}_i$, but the elements of $\hat{\boldsymbol{w}}_i$ are different from the ones of $\tilde{\boldsymbol{w}}_i$. The eigenvalues $\hat{\lambda}_i$'s of $\tilde{\boldsymbol{B}}\tilde{\boldsymbol{F}}^{-1}$ correspond to λ_i's of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$, because the eigenvectors obtained from (20) are the left eigenvectors of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$, corresponding to λ_i's. Moreover, the conjugately transposed vectors of the eigenvectors obtained from (20) correspond (or are equal) to the row vectors of $\tilde{\boldsymbol{H}}$ in (5) up to constants. The proof of the mentioned above is as follows: The matrix $\tilde{\boldsymbol{B}}$ in (18) can be decomposed as

$$\tilde{\boldsymbol{B}} = \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{H}}, \tag{21}$$

where $\tilde{\boldsymbol{\Lambda}}$ is the block diagonal matrix defined by

$$\tilde{\boldsymbol{\Lambda}} := \text{block-diag}\{\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \cdots, \boldsymbol{\Lambda}_n\}, \tag{22}$$

$$\boldsymbol{\Lambda}_i := \text{diag}\{\cdots, |a_i(-1)|^2\gamma_r, |a_i(0)|^2\gamma_i, |a_i(1)|^2\gamma_i, \cdots\} \tag{23}$$

$i = \overline{1,n}$. Substituting (13) and (21) into (20), we obtain

$$\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{H}}(\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Psi}}\tilde{\boldsymbol{H}})^{-1}\hat{\boldsymbol{w}}_i = \hat{\lambda}_i\hat{\boldsymbol{w}}_i. \tag{24}$$

It can be easily shown that $\tilde{\boldsymbol{H}}$ is nonsingular when $L_1 = -\infty$ and $L_2 = \infty$. Then (24) becomes

$$\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{\Psi}}^{-1}\tilde{\boldsymbol{H}}^{-H}\hat{\boldsymbol{w}}_i = \hat{\lambda}_i\hat{\boldsymbol{w}}_i, \tag{25}$$

Multiplying (25) by $\tilde{\boldsymbol{H}}^{-H}$ from the left side, (25) becomes

$$\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{\Psi}}^{-1}\tilde{\boldsymbol{H}}^{-H}\hat{\boldsymbol{w}}_i = \hat{\lambda}_i\tilde{\boldsymbol{H}}^{-H}\hat{\boldsymbol{w}}_i. \tag{26}$$

Let $\hat{\boldsymbol{g}}_i := \tilde{\boldsymbol{H}}^{-H}\hat{\boldsymbol{w}}_i$, then (26) becomes $\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{\Psi}}^{-1}\hat{\boldsymbol{g}}_i = \hat{\lambda}_i\hat{\boldsymbol{g}}_i$. This means that since $\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{\Psi}}^{-1}$ is a diagonal matrix, the elements of $\hat{\boldsymbol{g}}_i$ are zero except for one element. Namely, since all elements of $\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{\Psi}}^{-1}$ are distinct, it can be seen that the conjugately transposed vector of $\hat{\boldsymbol{w}}_i$ becomes a row vector of $\tilde{\boldsymbol{H}}$ up to a constant. The proof is completed.

It can be seen from the definition of the block element $\boldsymbol{H}_{ij}$ (see it stated below (7)) that $\boldsymbol{H}_{ij}$ is a matrix (of L columns and possibly infinite number of rows) having a special Toeplitz (or constant-along-diagonals) structure. Therefore, the (cross) correlation of a pair of rows of $\boldsymbol{H}_{ij}$ (by shifting their elements left or right appropriately) is the same for all pairs of rows of $\boldsymbol{H}_{ij}$ if L is infinite. In practice, however, the length L of the equalizer and the length K of the channel are finite, and so $\boldsymbol{H}_{ij}$ is a matrix of L columns and $L + K - 1$ rows, that is, $\boldsymbol{H}_{ij} \in \boldsymbol{C}^{(L+K-1)\times L}$. In this case pairs of rows of $\boldsymbol{H}_{ij}$ have approximately the similar correlations for all pairs of rows of $\boldsymbol{H}_{ij}$ if L is sufficiently large. Base on the above discussions, we can classify approximately nL eigenvectors $\tilde{\boldsymbol{w}}_i$'s in (18) into n sets of L eigenvectors whose pairs have almost the same correlations for all pairs of eigenvectors of each set. Thus we propose a tentative procedure of finding n appropriate eigenvectors satisfying (8) is as follows;

1) Set k = 1 (where k denotes the number of iterations from the beginning less than n+1).

2) Select the eigenvector $\tilde{\boldsymbol{w}}$ of $\tilde{\boldsymbol{F}}^{-1}\tilde{\boldsymbol{B}}$ and the eigenvector $\hat{\boldsymbol{w}}$ of $\tilde{\boldsymbol{B}}\tilde{\boldsymbol{F}}^{-1}$ corresponding to the maximum magnitude eigenvalue among $|\hat{\lambda}_i|$'s.

3) Calculate the magnitude of the correlations of all pairs of $\hat{\boldsymbol{w}}$ and $\hat{\boldsymbol{w}}_i$'s.

4) Separate L eigenvalues $\hat{\lambda}$'s from the others such that their magnitudes are larger than the remaining $(n-k)L$ ones, and save the $(n-k)L$ remaining eigenvalues $\hat{\lambda}$'s for finding other eigenvectors.

5) Put $k = k + 1$ and stock the $\tilde{\boldsymbol{w}}$ obtained in **2)**. If $k = n + 1$, stop the iterations, otherwise, go to **2)**.

Therefore, the n eigenvectors $\tilde{\boldsymbol{w}}$'s stocked in step **5)** are the n solutions in (8)

4. COMPUTER SIMULATIONS

To demonstrate the validity of the proposed method, many computer simulations were conducted. Some results are shown in this section. The unknown system $\boldsymbol{H}(z)$ was set to be an FIR channel with two inputs and two outputs, and assumed that the length of channel was three ($K = 3$), that is, $\boldsymbol{H}^{(k)}$'s in (1) were set to be $\boldsymbol{H}(z) = \sum_{k=0}^{2}\boldsymbol{H}^{(k)}z^k =$

$$\begin{bmatrix} 1.00 + 0.15z + 0.10z^2 & 0.65 + 0.25z + 0.15z^2 \\ 0.50 - 0.10z + 0.20z^2 & 1.00 + 0.25z + 0.10z^2 \end{bmatrix}.$$

The Gaussian noises $n_j(t)$ with its variance $\sigma^2_{n_j}$ were included in the output $y_j(t)$ at various SNR levels. The SNR was considered at the output of the system $\boldsymbol{H}(z)$. The source signals $s_1(t)$ and $s_2(t)$ were a sub-Gaussian signal and a super-Gaussian signal, where the sub-Gaussian signal takes one of two values, -1 and 1 with equal probability 1/2 and the super-Gaussian signal takes -2, 2, and 0 with probabilities 1/8, 1/8, and 6/8, respectively. As a measure of performances, we used the $multichannel\ intersymbol\ interference$ ($\mathrm{M_{ISI}}$) [3]. The parameters L_1 and L_2 in $\boldsymbol{W}(z)$ were set to be 0 and 9, respectively. The first and the second components of the reference system $\boldsymbol{f}(z)$ were, respectively, set to be z^2 and 0. that is, $x(t) = y_1(t-2)$. For comparison, the EVA was used.

Fig. 2 shows the results of performances of the proposed REVA and the EVA when the SNR levels were respectively taken to be 5 [dB], 10 [dB], 15 [dB], 20 [dB], and 25 [dB], where each $\mathrm{M_{ISI}}$ shown in Fig. 2 was the average of the performances obtained by 30 independent Monte Carlo runs. In each Monte Carlo run, the final eigenvectors of the EVA and the REVA were obtained by ten iterative calculations, where in each iteration, $\tilde{\boldsymbol{F}}$ and $\tilde{\boldsymbol{B}}$ were estimated by data samples in the following three cases; (Case 1) 5,000 samples, (Case 2) 10,000 samples, and (Case 3) 20,000 samples. It can be seen from Fig. 2 that when the SNR level is more than about 15 dB, the EVA is more useful than the REVA, because at those SNR levels, the EVA can provide better performances than the REVA. On the other hand, the REVA is effective for the case that the SNR level is less than about 15 dB, because as the number of data samples increases, the REVA provides better performances than the EVA.

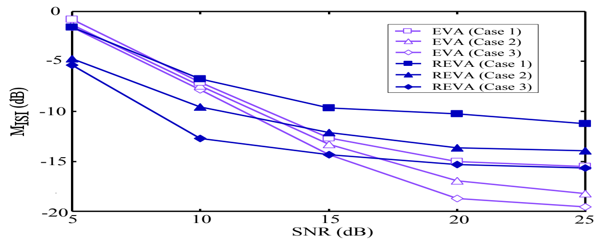

Fig. 2. The performances of the REVA and the EVA with varying SNR levels, in the cases of 5,000 samples (Case 1), 10,000 samples (Case 2), and 20,000 samples (Case 3).

5. CONCLUSIONS

We have proposed an EVA for solving the BD problem. The EVA is robust against Gaussian noise, which means that the EVA can be used to estimate the inverse of $\tilde{\boldsymbol{H}}$ with as little influence of Gaussian noise as possible. Computer simulations have demonstrated the effectiveness of the proposed EVA.

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