Recently Developed Approaches for Solving Blind Deconvolution of MIMO-IIR Systems: Super-Exponential and Eigenvector Methods

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Abstract— **Recently we develop two kinds of approaches, that is, the super-exponential method (SEM) and the eigenvector method (EVM), of which both can achieve the blind deconvolution for MIMO-IIR systems. It is shown that these methods are closely related each other. Based on this fact, we propose a new SEM incorporated with the EVM. Simulation results will be presented for showing the validity of the proposed method.**

I. Introduction

In this paper, we deal with a blind deconvolution (BD) problem for a multiple-input and multiple-output (MIMO) infinite-impulse response (IIR) channels. A large number of methods for solving the BD problem have been proposed until now (see [1], and reference therein). This paper focuses on two methods, that is, the Super-exponential method (SEM) and the Eigenvector method (EVM) to solve the BD problem.

Shalvi and Weinstein [8] have proposed the SEM for the first time. One of the attractive properties of the SEM is that the SEM converges to a desired solution at a super-exponential rate and its global convergence to the solution is guaranteed. However, deflation methods are needed to recover all source signals. As for the EVM, the first proposal of the EVM was done by Jelonnek et al. [4]. They have proposed the EVM for solving blind equalization (BE) problems of single-input single-output (SISO) channels and single-input multiple-output (SIMO) channels. The most attractive feature of the EVM is that its algorithm can be derived from a closed-form solution using reference signals. Owing to the property, differently from the algorithms derived from steepest descent methods, the EVM does not need many iterations to achieve the BE, but works so as to solve the BE problem with one iteration. Recently, we extended the EVM to the case of MIMO-IIR channels [6]. Then we proved that the proposed EVM can work so as to recover all source signals from their mixtures with one iteration. However, the EVM in [6] has different performances for a different choice of the reference signal (see section V).

This paper proposes a new algorithm which combines the SEM and the EVM in [6] so that their superiorities in performance are incorporated. To this end, we analyze

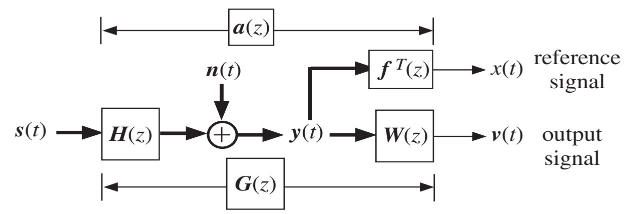

Fig. 1. The composite system of an unknown system and a deconvolver, and a reference system.

the relationship between the SEM and EVM. Based on the analysis, we propose such a two-stage algorithm that at first the EVM estimates roughly a solution for achieving the BD, and then using the estimated solution, the SEM works so that the solution can be obtained with as good accuracy as possible, compared with the EVM in [6]. Simulation results show a performance of superiority of the proposed algorithm.

The present paper uses the following notation: Let Z denote the set of all integers. Let C denote the set of all complex numbers. Let $\boldsymbol{C}^n$ denote the set of all n-column vectors with complex components. Let $\boldsymbol{C}^{m\times n}$ denote the set of all $m \times n$ matrices with complex components. The superscripts T, $*$, and H denote, respectively, the transpose, the complex conjugate, and the complex conjugate transpose (Hermitian) of a matrix. The symbols block-diag$\{\cdots\}$ and diag$\{\cdots\}$ denote respectively a block diagonal and a diagonal matrices with the block diagonal and the diagonal elements $\{\cdots\}$. The symbol cum$\{x_1,x_2,x_3,x_4\}$ denotes the fourth-order cumulant of x_i's. Let $i = \overline{1,n}$ stand for $i = 1, 2, \cdots, n$.

II. Problem Formulation and Assumptions

We consider a MIMO system with n inputs and m outputs as described by

$$\boldsymbol{y}(t) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k) + \boldsymbol{n}(t), \quad t \in Z, \qquad (1)$$

where $\boldsymbol{s}(t)$ is an n-column vector of input (or source) signals, $\boldsymbol{y}(t)$ is an m-column vector of system outputs, $\boldsymbol{n}(t)$ is an m-column vector of Gaussian noises, and $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ impulse response matrix sequence. The transfer function of the system is defined by $\boldsymbol{H}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} z^k, z \in C$.

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To recover the source signals, we process the output signals by an $n \times m$ deconvolver (or equalizer) $\boldsymbol{W}(z)$ described by

$$\begin{aligned}\boldsymbol{v}(t) &= \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{y}(t-k) \\ &= \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} \boldsymbol{s}(t-k) + \sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{n}(t-k), \end{aligned} \quad (2)$$

where $\{\boldsymbol{G}^{(k)}\}$ is the impulse response matrix sequence of $\boldsymbol{G}(z) := \boldsymbol{W}(z)\boldsymbol{H}(z)$, which is defined by $\boldsymbol{G}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} z^k, z \in C$. The cascade connection of the unknown system and the deconvolver is illustrated in Fig. 1.

Here, we put the following assumptions on the system, the source signals, the deconvolver, and the noises.

A1) The transfer function $\boldsymbol{H}(z)$ is stable and has full column rank on the unit circle $|z| = 1$, where the assumption **A1)** implies that the unknown system has less inputs than outputs, i.e., $n \le m$, and there exists a left stable inverse of the unknown system.

A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a complex, zero-mean and non-Gaussian random vector process with element processes $\{s_i(t)\}$, $i = \overline{1,n}$ being mutually independent. Each element process $\{s_i(t)\}$ is an i.i.d. process with a variance $\sigma_{s_i}^2 \neq 0$ and a nonzero fourth-order cumulant $\gamma_i \neq 0$ defined as

$$\gamma_i = \mathrm{cum}\{s_i(t), s_i(t), s_i^*(t), s_i^*(t)\} \neq 0. \quad (3)$$

A3) The deconvolver $\boldsymbol{W}(z)$ is an FIR system, that is, $\boldsymbol{W}(z) = \sum_{L_1}^{L_2} \boldsymbol{W}^{(k)} z^k$, where the length $L := L_2 - L_1 + 1$ is taken to be sufficiently large so that the truncation effect can be ignored.

A4) The noise sequence $\{\boldsymbol{n}(t)\}$ is a zero-mean, Gaussian vector stationary process whose component processes $\{n_j(t)\}$, $j = \overline{1,m}$ have nonzero variances $\sigma_{n_j}^2$, $j = \overline{1,m}$.

A5) The two vector sequences $\{\boldsymbol{n}(t)\}$ and $\{\boldsymbol{s}(t)\}$ are mutually statistically independent.

Under **A3)**, the impulse response $\{\boldsymbol{G}^{(k)}\}$ of the cascade system is given by

$$\boldsymbol{G}^{(k)} := \textstyle\sum_{\tau=L_1}^{L_2} \boldsymbol{W}^{(\tau)} \boldsymbol{H}^{(k-\tau)}, \quad k \in Z, \quad (4)$$

In a vector form, (4) can be written as

$$\tilde{\boldsymbol{g}}_i = \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i, \quad i = \overline{1,n}, \quad (5)$$

where $\tilde{\boldsymbol{g}}_i$ is the column vector consisting of the ith output impulse response of the cascade system defined by $\tilde{\boldsymbol{g}}_i := [\boldsymbol{g}_{i1}^T, \boldsymbol{g}_{i2}^T, \cdots, \boldsymbol{g}_{in}^T]^T$,

$$\boldsymbol{g}_{ij} := [\cdots, g_{ij}(-1), g_{ij}(0), g_{ij}(1), \cdots]^T, \quad j = \overline{1,n} \quad (6)$$

where $g_{ij}(k)$ is the (i,j)th element of matrix $\boldsymbol{G}^{(k)}$, and $\tilde{\boldsymbol{w}}_i$ is the mL-column vector consisting of the tap coefficients (corresponding to the ith output) of the deconvolver defined by $\tilde{\boldsymbol{w}}_i := \left[\boldsymbol{w}_{i1}^T, \boldsymbol{w}_{i2}^T, \cdots, \boldsymbol{w}_{im}^T\right]^T \in \boldsymbol{C}^{mL}$,

$$\boldsymbol{w}_{ij} := [w_{ij}(L_1), w_{ij}(L_1+1), \cdots, w_{ij}(L_2)]^T \in \boldsymbol{C}^L, \quad (7)$$

$j = \overline{1,m}$, where $w_{ij}(k)$ is the (i,j)th element of matrix $\boldsymbol{W}^{(k)}$, and $\tilde{\boldsymbol{H}}$ is the $n \times m$ block matrix whose (i,j)th block element $\boldsymbol{H}_{ij}$ is the matrix (of L columns and possibly infinite number of rows) with the (l,r)th element $[\boldsymbol{H}_{ij}]_{lr}$ defined by $[\boldsymbol{H}_{ij}]_{lr} := h_{ji}(l-r)$, $l = 0, \pm 1, \pm 2, \cdots$, $r = \overline{L_1, L_2}$, where $h_{ij}(k)$ is the (i,j)th element of the matrix $\boldsymbol{H}^{(k)}$.

In the MIMO deconvolution problem, we want to adjust $\tilde{\boldsymbol{w}}_i$'s (i = $\overline{1,n}$) so that

$$[\tilde{\boldsymbol{g}}_1, \cdots, \tilde{\boldsymbol{g}}_n] = \tilde{\boldsymbol{H}}[\tilde{\boldsymbol{w}}_1, \cdots, \tilde{\boldsymbol{w}}_n] = [\tilde{\boldsymbol{\delta}}_1, \cdots, \tilde{\boldsymbol{\delta}}_n]\boldsymbol{P}, \quad (8)$$

where $\boldsymbol{P}$ is an $n \times n$ permutation matrix, and $\tilde{\boldsymbol{\delta}}_i$ is the n-block column vector defined by $\tilde{\boldsymbol{\delta}}_i := [\boldsymbol{\delta}_{i1}^T, \boldsymbol{\delta}_{i2}^T, \ldots, \boldsymbol{\delta}_{in}^T]^T$, $i = \overline{1,n}$, $\boldsymbol{\delta}_{ij} := \hat{\boldsymbol{\delta}}_i$, for $i = j$, otherwise $(\cdots, 0, 0, 0, \cdots)^T$. Here, $\hat{\boldsymbol{\delta}}_i$ is the column vector (of infinite elements) whose rth element $\hat{\delta}_i(r)$ is given by $\hat{\delta}_i(r) = d_i \delta(r - k_i)$, where $\delta(t)$ is the Kronecker delta function, d_i is a complex number standing for a scale change and a phase shift, and k_i is an integer standing for a time shift.

III. Relationship between the SEM and the EVM

Jelonnek et al. [4] have shown in the single-input case that from the following problem, that is,

$$\begin{aligned} &\text{Maximize } D_{v_i x} = \mathrm{cum}\{v_i(t), v_i^*(t), x(t), x^*(t)\} \\ &\text{under } \sigma_{v_i}^2 = \sigma_{s_{\rho_i}}^2, \end{aligned} \quad (9)$$

a closed-form solution expressed as a generalized eigenvector problem can be led by the Lagrangian method, where $\sigma_{v_i}^2$ and $\sigma_{s_{\rho_i}}^2$ denote the variances of the output $v_i(t)$ and a source signal $s_{\rho_i}(t)$, respectively, ρ_i is one of integers $\{1, 2, \cdots, n\}$ such that the set $\{\rho_1, \rho_2, \cdots, \rho_n\}$ is a permutation of the set $\{1, 2, \cdots, n\}$, $v_i(t)$ is the ith element of $\boldsymbol{v}(t)$ in (2), and the reference signal $x(t)$ is given by $\boldsymbol{f}^T(z)\boldsymbol{y}(t)$ using an appropriate filter $\boldsymbol{f}(z)$ (see Fig. 1). The filter $\boldsymbol{f}(z)$ is called a *reference system.* Let $\boldsymbol{a}(z) := \boldsymbol{H}^T(z)\boldsymbol{f}(z) = [a_1(z), a_2(z), \cdots, a_n(z)]^T$, then $x(t) = \boldsymbol{f}^T(z)\boldsymbol{H}(z)\boldsymbol{s}(t) = \boldsymbol{a}^T(z)\boldsymbol{s}(t)$. The element $a_i(z)$ of the filter $\boldsymbol{a}(z)$ is defined as $a_i(z) = \sum_{k=-\infty}^{\infty} a_i(k) z^k$ and the reference system $\boldsymbol{f}(z)$ is an m-column vector whose elements are $f_j(z) = \sum_{k=L_1}^{L_2} f_j(k) z^k$, $j = \overline{1,m}$.

In our case, $D_{v_i x}$ and $\sigma_{v_i}^2$ can be expressed in terms of the vector $\tilde{\boldsymbol{w}}_i$ as, respectively, $D_{v_i x} = \tilde{\boldsymbol{w}}_i^H \tilde{\boldsymbol{B}} \tilde{\boldsymbol{w}}_i$ and $\sigma_{v_i}^2 = \tilde{\boldsymbol{w}}_i^H \tilde{\boldsymbol{R}} \tilde{\boldsymbol{w}}_i$, where $\tilde{\boldsymbol{B}}$ is the $m \times m$ block matrix whose (i,j)th block element $\boldsymbol{B}_{ij}$ is the matrix with the (l,r)th element $[\boldsymbol{B}_{ij}]_{lr}$ calculated by $\mathrm{cum}\{y_i^*(t\text{-}L_1\text{-}l\text{+}1), y_j(t\text{-}L_1\text{-}r\text{+}1), x^*(t), x(t)\}$ $(l, r = \overline{1,L})$ and $\tilde{\boldsymbol{R}} = E[\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)]$ is the covariance matrix of m-block column vector $\tilde{\boldsymbol{y}}(t)$ defined by

$$\tilde{\boldsymbol{y}}(t) := \left[\boldsymbol{y}_1^T(t), \boldsymbol{y}_2^T(t), \cdots, \boldsymbol{y}_m^T(t)\right]^T \in \boldsymbol{C}^{mL}, \quad (10)$$

where $\boldsymbol{y}_j(t) := [y_j(t\text{-}L_1), y_j(t\text{-}L_1\text{-}1), \cdots, y_j(t\text{-}L_2)]^T \in \boldsymbol{C}^L$, $j = \overline{1,m}$. It follows from (10) that $\tilde{\boldsymbol{y}}(t)$ is expressed as $\tilde{\boldsymbol{y}}(t) = \boldsymbol{D}_c(z)\boldsymbol{y}(t)$, where $\boldsymbol{D}_c(z)$ is an $mL \times m$ converter (consisting of m identical delay chains each of which has L delay elements when $L_1 = 1$) defined by $\boldsymbol{D}_c(z) :=$ block-diag$\{\boldsymbol{d}_c(z), \cdots, \boldsymbol{d}_c(z)\}$ with m diagonal block elements all being the same L-column vector $\boldsymbol{d}_c(z)$ defined by $\boldsymbol{d}_c(z) = [z^{L_1}, \cdots, z^{L_2}]^T$. Therefore, by the similar way to as in [4], the maximization of $|D_{v_i x}|$ under $\sigma_{v_i}^2 = \sigma_{s_{\rho_i}}^2$ leads to the following generalized eigenvector problem;

$$\tilde{\boldsymbol{B}} \tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{R}} \tilde{\boldsymbol{w}}_i. \quad (11)$$

Moreover, Jelonnek et al. have shown in [4] that the eigenvector corresponding to the maximum magnitude eigenvalue of $\tilde{\boldsymbol{R}}^\dagger \tilde{\boldsymbol{B}}$ becomes the solution of the blind equalization problem, which is referred to as an *eigenvector algorithm* (EVA). It has been also shown in [6] that the BD for MIMO-IIR systems can be achieved with the eigenvectors of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}$, using only one reference signal. Note that since Jelonnek et al. have dealt with SISO-IIR systems or SIMO-IIR systems, the constructions of $\tilde{\boldsymbol{B}}$, $\tilde{\boldsymbol{w}}_i$, and $\tilde{\boldsymbol{R}}$ in (11) are different from those proposed in [4].

Castella et al. [2] have shown that from (9), a BD can be iteratively achieved by using $x_i(t) = \tilde{\boldsymbol{w}}_i\tilde{\boldsymbol{y}}(t)$ $(i = \overline{1,n})$ as reference signals (see Fig. 2), where the number of reference signals corresponds to the number of source signals and $\tilde{\boldsymbol{w}}_i$ is an eigenvector obtained by $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i$ in the previous iteration, where $\tilde{\boldsymbol{B}}_i$ represents $\tilde{\boldsymbol{B}}$ in (11) calculated by $x_i(t) = \tilde{\boldsymbol{w}}_i\tilde{\boldsymbol{y}}(t)$. Then a deflation method was used to recover all source signals.

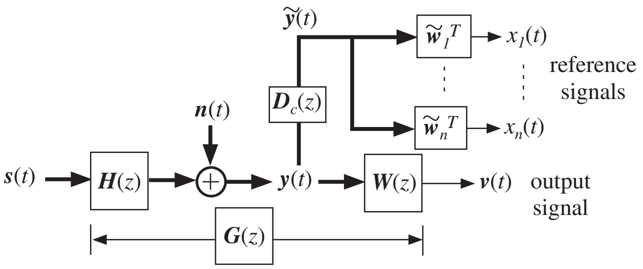

Fig. 2. The composite system of an unknown system and a deconvolver, and a reference system.

Let $\tilde{\boldsymbol{d}}_i$ denote $\tilde{\boldsymbol{B}}_i\tilde{\boldsymbol{w}}_i$. Then the EVM in [2] can be expressed as

$$\tilde{\boldsymbol{w}}_i = \frac{1}{\lambda_i}\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{d}}_i, \;\; i = \overline{1,n}. \tag{12}$$

Differently from the EVM in [2], (12) means that $\tilde{\boldsymbol{w}}_i$ is modified iteratively by the value of the right-hand side of (12) without calculating the eigenvectors of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i$, where $\tilde{\boldsymbol{w}}_i$ in both $x_i(t)$ and $\tilde{\boldsymbol{d}}_i$ is the value of the left-hand side of (12) in the previous iteration. The scalar λ_i is fixed to be 1, but $\tilde{\boldsymbol{w}}_i$ obtained by (12) should be normalized at each iteration, that is,

$$\tilde{\boldsymbol{w}}_i := \frac{\tilde{\boldsymbol{w}}_i}{\sqrt{\tilde{\boldsymbol{w}}_i^H\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i}}, \;\; i = \overline{1,n}. \tag{13}$$

It can be seen below that the iterative algorithm (12) is nothing but an iterative procedure of the SEM: The matrices $\tilde{\boldsymbol{R}}$ and $\tilde{\boldsymbol{B}}_i$ can be expressed as

$$\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{H}}, \;\; \tilde{\boldsymbol{B}}_i = \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Lambda}}_i\tilde{\boldsymbol{H}}, \tag{14}$$

where $\tilde{\boldsymbol{\Sigma}}$ is the block diagonal matrix defined by $\tilde{\boldsymbol{\Sigma}}$:= block-diag$\{\boldsymbol{\Sigma}_1,\boldsymbol{\Sigma}_2,\cdots,\boldsymbol{\Sigma}_n\}$, $\boldsymbol{\Sigma}_i$:= diag$\{\cdots, \sigma_{s_i}^2,\sigma_{s_i}^2,\sigma_{s_i}^2,\cdots\}$, $i = \overline{1,n}$, and $\tilde{\boldsymbol{\Lambda}}_i$ is the block diagonal matrix defined by $\tilde{\boldsymbol{\Lambda}}_i$:= block-diag$\{\boldsymbol{\Lambda}_{i1},\boldsymbol{\Lambda}_{i2},\cdots,\boldsymbol{\Lambda}_{in}\}$,

$$\boldsymbol{\Lambda}_{ij} := \text{diag}\{\cdots, |g_{ij}(\text{-}1)|^2\gamma_j, |g_{ij}(0)|^2\gamma_j, |g_{ij}(1)|^2\gamma_j, \cdots\}, \tag{15}$$

$j = \overline{1,n}$. Then, from (5) and (14), $\tilde{\boldsymbol{d}}_i = \tilde{\boldsymbol{B}}_i\tilde{\boldsymbol{w}}_i$ can be written as

$$\tilde{\boldsymbol{d}}_i = \tilde{\boldsymbol{B}}_i\tilde{\boldsymbol{w}}_i = \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Lambda}}_i\tilde{\boldsymbol{g}}_i, \tag{16}$$

From (15) and (16), one can see that the elements of $\tilde{\boldsymbol{\Lambda}}_i\tilde{\boldsymbol{g}}_i$ are $|g_{ij}(k)|^2 g_{ij}(k)\gamma_j$, $k = \overline{-\infty,\infty}$. Let us define

$$f_{ij}(k) = \frac{\gamma_j}{\sigma_{s_j}^2}|g_{ij}(k)|^2 g_{ij}(k). \tag{17}$$

This is the SEM with respect to $g_{ij}(k)$, using the fourth-order cumulants [3]. Substituting (17) into (16), we obtain

$$\tilde{\boldsymbol{d}}_i = \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{f}}_i, \tag{18}$$

where $\tilde{\boldsymbol{f}}_i = [\boldsymbol{f}_{i1}^T,\boldsymbol{f}_{i2}^T,\cdots,\boldsymbol{f}_{in}^T]^T = [\cdots,f_{ij}(\text{-}1),\; f_{ij}(0),f_{ij}(1),\cdots]^T$. Moreover, substituting (14) and (18) into (12), we have

$$\tilde{\boldsymbol{w}}_i = (\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{H}})^{\dagger}\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{f}}_i, \tag{19}$$

where $\lambda_i = 1$. (19) is the SEM with respect to $\tilde{\boldsymbol{w}}_i$ [3]. This completes the proof.

Therefore, the SEM can be derived from the following problem:

$$\begin{aligned} &\text{Maximize } D_{v_i x_i} = \text{cum}\{v_i(t), x_i^*(t), x_i(t), x_i^*(t)\} \\ &\text{under } \sigma_{v_i}^2 = \sigma_{s_{\rho_i}}^2, \end{aligned} \tag{20}$$

where $x_i(t) = \tilde{\boldsymbol{w}}_i\tilde{\boldsymbol{y}}(t)$ $(i = \overline{1,n})$ and $\tilde{\boldsymbol{w}}_i$ in $x_i(t)$ is the value of the left-hand side (12) in the previous iteration. The proof of the mentioned above is omitted, because of the page limitation.

IV. The Proposed Algorithm

In this paper, we consider an effect of combining (11) with (12). That is, since the initial values of $\tilde{\boldsymbol{w}}_i$'s of estimating $\tilde{\boldsymbol{d}}_i$'s are needed to work (12), $\tilde{\boldsymbol{w}}_i$'s obtained by (11) with an appropriate reference signal $x(t)$ are used as the initial values for estimating $\tilde{\boldsymbol{d}}_i$'s of (12). Because the EVM in (11) can work so as to recover all source signals, using any reference signal $x(t)$, then if the $\tilde{\boldsymbol{w}}_i$'s obtained by the EVM are used to estimate $\tilde{\boldsymbol{d}}_i$'s of (12), the SEM in (12) does not need deflation methods to recover all source signals. Because the $\tilde{\boldsymbol{w}}_i$'s obtained by the EVM are linearly independent. Therefore, we propose the following two-stage algorithm:

Stage 1) Roughly calculate the eigenvectors of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}$ with an appropriate reference signal $x(t)$ in (9).
Using the calculated eigenvectors as initial values of $\tilde{\boldsymbol{w}}_i$'s of estimating $\tilde{\boldsymbol{d}}_i$'s,

Stage 2) Carry out the iterative procedure, of which each iteration consists of two steps (12) with $\lambda_i = 1$ and (13).

Remark 1: We can expect that the proposed two-stage algorithm overcomes the drawbacks of the EVM described in Section I, because it can globally converge to a solution in (8).

The procedure of the proposed two-stage algorithm is summarized as follows: Choose an appropriate reference signal $x(t)$ and appropriate initial values of $\tilde{\boldsymbol{w}}_i^{[1]}(0)$, $\tilde{\boldsymbol{R}}(0)$, $\tilde{\boldsymbol{B}}(0)$, $\tilde{\boldsymbol{B}}_i(0)$

for $t_l = 1 : t_{l_{all}}$
  if $t_l < t_s$
    for $t = t_d(t_l - 1)$+1:$t_d t_l$
      Estimate $\tilde{\boldsymbol{R}}(t)$ and $\tilde{\boldsymbol{B}}(t)$ by their moving averages.
    end
    Calculate the eigenvectors $\tilde{\boldsymbol{w}}_i^{[1]}(t_l)$'s from $\tilde{\boldsymbol{R}}^{\dagger}(t)\tilde{\boldsymbol{B}}(t)$ (Stage 1)).
  elseif $t_l \geq t_s$
    if $t_l == t_s$, $\tilde{\boldsymbol{w}}_i^{[2]}(t_l - 1) = \tilde{\boldsymbol{w}}_i^{[1]}(t_s - 1)$
    for $t = t_d(t_l - 1)$+1:$t_d t_l$

$x_i(t) = \tilde{\boldsymbol{w}}_i^{[2]T}(t_l - 1)\tilde{\boldsymbol{y}}_i(t)$
Estimate $\tilde{\boldsymbol{R}}(t)$ and $\tilde{\boldsymbol{B}}_i(t)$ by their moving averages.
end
Calculate the vector $\tilde{\boldsymbol{w}}_i^{[2]}(t_l)$ with (12) (Stage 2)).
end
end

Here, $t_{l_{all}}$ denotes the total number of iterations and t_d denotes the number of data samples for estimating the matrices $\tilde{\boldsymbol{R}}(t)$, $\tilde{\boldsymbol{B}}(t)$, and $\tilde{\boldsymbol{B}}_i(t)$. $\tilde{\boldsymbol{w}}_i^{[1]}(t)$ and $\tilde{\boldsymbol{w}}_i^{[2]}(t)$ are the eigenvectors obtained for Stage 1) and the vectors obtained for Stage 2), respectively. t_s denotes an arbitrary integer satisfying $2 < t_s < t_{l_all}$. For $0 < t_l < t_s$ and $t_s \leq t_l \leq t_{l_{all}}$, the eigenvectors $\tilde{\boldsymbol{w}}_i^{[1]}(t_l)$'s and the vectors $\tilde{\boldsymbol{w}}_i^{[2]}(t_l)$'s are iteratively calculated, respectively, according to Stage 1) and Stage 2).

V. Computer simulations

To demonstrate the validity of the proposed algorithm, many computer simulations were conducted. Some results are shown in this section. The unknown system $\boldsymbol{H}(z)$ was set to be the same system with two inputs and three outputs as in [6]. The source signals $s_1(t)$ and $s_2(t)$ were a sub-Gaussian signal which takes one of two values, -1 and 1 with equal probability 1/2. The Gaussian noises $n_j(t)$ with its variance $\sigma_{n_j}^2$ were included in the output $y_j(t)$ at various SNR levels. The SNR was considered at the output of the system $\boldsymbol{H}(z)$. The parameters L_1 and L_2 in $\boldsymbol{W}(z)$ were set to be 0 and 11, respectively. As a measure of performances, we used the $multichannel\ intersymbol\ interference$ ($\mathrm{M_{ISI}}$) [7], which was the average of 50 Monte Carlo runs. In each Monte Carlo run, the number of iterations $t_{l_{all}}$ was set to be 10, the number of data samples t_d was set to be 5,000, and the threshold t_s was set to be 6.

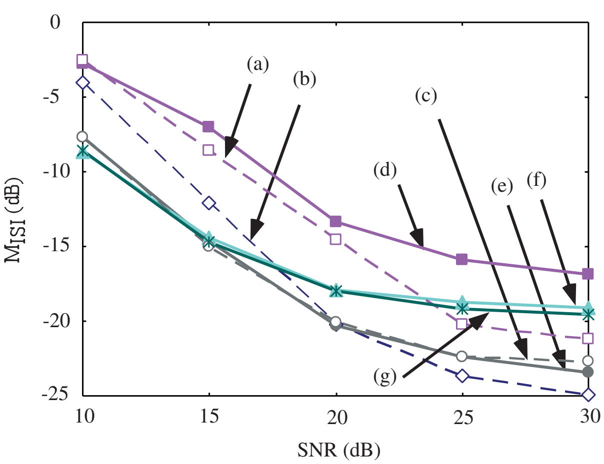

Fig. 3. The performances of the proposed algorithm and the conventional EVMs and two-stage SEM at different SNR levels, in the cases of 5,000 data samples.

The curves (a), (b) and (c) represent the performances obtained by our proposed two-stage algorithm, where in the EVM in the stage 1), $x(t) = \sum_{i=1}^{3} f_i(2)y_i(t-2)$, $\sum_{i=1}^{2} f_i(3)y_i(t-3)$, and $f_2(2)y_2(t-2)$ were chosen as reference signals, which correspond to (a), (b), and (c), respectively, and $f_i(2)$'s and $f_i(3)$'s were randomly chosen from a Gaussian distribution with zero mean and unit variance. The curves (d) and (e) represent the performances obtained by only the EVA (t_s = 11) with respectively $x(t) = \sum_{i=1}^{3} f_i(2)y_i(t-2)$ and $f_2(2)y_2(t-2)$ as the reference signal. The curve (f) represents the performance obtained by Castella et al. (CRMPA). Finally the curve (g) is the performance obtained by the two-stage SEM in [3].

From the curves (d) and (e), one can see that the EVA's performance depends on the choice of the reference signal. From the curves (a), (b), and (c), it can be seen that for the low noise levels (more than 25 dB), the affect of the reference choice of the EVM in the stage 1) is reduced, but for the high noise levels (less than 20 dB), as the reference signal is complicated, the performances become worse. Moreover, the performances obtained by the proposed algorithm are better than the one of the SEM in the case of low noise levels (more than 25 dB). From the curve (c), for the case of less than 15 dB, the proposed algorithm is almost same performance as the SEM, but for the case of more than 15 dB, the performance obtained by the proposed algorithm is better than the SEM. Therefore from all the results we conclude that an advantage of combining the SEM with the EVM is shown, especially when the noise level is low, and when one use simple reference signals, e.g., $x(t) = f_2(2)y_2(t-2)$.

VI. Conclusion

We have proposed a two-stage algorithm for solving the BD problem which combines the SEM and the EVM so that their superiorities in performance are incorporated. Moreover, we have clarified the relationship between the SEM and the EVM. The simulation results have demonstrated the effectiveness of the proposed algorithm.

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