ADAPTIVE SUPER-EXPONENTIAL ALGORITHMS FOR BLIND DECONVOLUTION OF MIMO SYSTEMS

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ABSTRACT

Multichannel blind deconvolution of finite-impulse response (FIR) or infinite-impulse response (IIR) systems is investigated using the multichannel super-exponential method. First, some properties are shown for the rank of the correlation matrices relevant to the multichannel super-exponential method. Then, the matrix inversion lemma is extended to the degenerate rank case. Based on these results, two types of adaptive multichannel super-exponential algorithms are presented, that is, the one in covariance form and the other in QR-factorization form.

1. INTRODUCTION

Multichannel blind deconvolution has recently received attention in such field as digital communications, image processing and neural information processing.

Recently, Shalvi and Weinstein proposed an attractive approach to single-channel blind deconvolution called the *super-exponential method* (SEM) [1]. Extensions of their idea to multichannel deconvolution were presented by Inouye and Tanebe [2], Martone [3], [4], and Yeung and Yau [5]. In particular, Martone [3] proposed an adaptive version of the SEM based on low-rank processing [6], but the Martone algorithm require a rank-revealing technique, while the present paper presents an explicit formula for revealing the rank of relevant correlation matrices in the absent of noise.

In the present paper, we show some properties of the rank of the relevant correlation matrices, and present a matrix pseudo-inversion lemma. Based on these results, we propose two type of adaptive multichannel super-exponential algorithms (AMSEA's), the one in covariance (correlation or Kalman-filter) form and the other in QR-factorization form.

The present paper uses the following notation: Let Z denote the set of all integers. Let $\boldsymbol{C}^{m\times n}$ denote the set of all $m\times n$ matrices with complex components. The superscripts T, $*$, H and $\dagger$ denote, respectively, the transpose, the complex conjugate, the complex conjugate transpose (Hermitian) and the (Moore-Penrose) pseudoinverse operations of a matrix. Let $i=\overline{1,n}$ stand for $i=1,2,\cdots,n$.

2. ASSUMPTIONS AND PRELIMINARIES

We consider an MIMO channel system with n inputs and m outputs as described by

$$\boldsymbol{y}(t)=\sum_{k=-\infty}^{\infty}\boldsymbol{H}^{(k)}\boldsymbol{s}(t-k),\qquad t\in Z,\tag{1}$$

where
- $\boldsymbol{s}(t)$ n-column vector of input (or source) signals,
- $\boldsymbol{y}(t)$ m-column vector of channel outputs,
- $\boldsymbol{H}^{(k)}$ $m\times n$ matrix of impulse responses.

The transfer function of the channel system is defined by

$$\boldsymbol{H}(z)=\sum_{k=-\infty}^{\infty}\boldsymbol{H}^{(k)}z^{k},\qquad z\in C.\tag{2}$$

For the time being, it is assumed for theoretical analysis that noise is absent in (1).

To recover the source signals, we process the output signals by an $n\times m$ equalizer (or deconvolver) $\boldsymbol{W}(z)$ described by

$$\boldsymbol{z}(t)=\sum_{k=-\infty}^{\infty}\boldsymbol{W}^{(k)}\boldsymbol{y}(t-k),\qquad t\in Z.\tag{3}$$

The objective of multichannel blind deconvolution is to construct an equalizer that recovers the original source signals only from the measurements of the corresponding outputs.

We put the following assumptions on the systems and the source signals.

A1) The transfer function $\boldsymbol{H}(z)$ is stable and has full column rank on the unit circle $|z|=1$ [this implies that the unknown system has less inputs than outputs, i.e., $n\le m$, and there exists a left stable inverse of the unknown system].

A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a complex, zero-mean, non-Gaussian random vector process with element processes $\{s_i(t)\}$, $i=\overline{1,n}$ being mutually independent. Moreover, each element process $\{s_i(t)\}$ is an i.i.d. process with a nonzero variance σ_i^2 and a nonzero fourth-order cumulant γ_i. The variances σ_i^2's and the fourth-order cumulants γ_i's are unknown.

A3) The equalizer $\boldsymbol{W}(z)$ is an FIR system of sufficient length L so that the truncation effect can be ignored.

Remark 1: As to A1), if the channel system $\boldsymbol{H}(z)$ is FIR, then a condition of the existence of an FIR equalizer is rank$\boldsymbol{H}(z) = n$ for all nonzero $z \in C$ [7]. Moreover, if $\boldsymbol{H}(z)$ is irreducible, then there exists an equalizer $\boldsymbol{W}(z)$ of length $L \le nK$, where K is the length of the channel system [7]. Besides, it is shown that there exists generically (or except for pathological cases) an equalizer $\boldsymbol{W}(z)$ of length $L \le \lceil \frac{nK}{m-n} \rceil$, where $\lceil x \rceil$ stands for the smallest integer that is greater than equal to x.

For now, there are two approaches to multichannel (or MIMO) blind deconvolution, a *concurrent blind deconvolution approach* and a *deflationary blind deconvolution approach*. The former is to deconvolve (or recover) concurrently all the source signals, while the latter is to deconvolve sequentially (or iteratively with respect to sources) the source signals one by one. The former approach is employed in the present paper and the latter approach will be developed in a forthcoming paper.

Let us consider an FIR equalizer with the transfer function $\boldsymbol{W}(z)$ given by

$$\boldsymbol{W}(z) = \sum_{k=L_1}^{L_2} \boldsymbol{W}^{(k)} z^k, \tag{4}$$

where the length $L{:=}L_2 - L_1 + 1$ is taken to be sufficiently large. Let $\tilde{\boldsymbol{w}}_i$ be the Lm-column vector consisting of the tap coefficient (corresponding to the ith output) of the equalizer defined by

$$\tilde{\boldsymbol{w}}_i := \left[\boldsymbol{w}_{i,1}^T, \boldsymbol{w}_{i,2}^T, \cdots, \boldsymbol{w}_{i,m}^T\right]^T \in \boldsymbol{C}^{mL}, \tag{5}$$

$$\boldsymbol{w}_{i,j} = \left[w_{i,j}{}^{(L_1)}, w_{i,j}{}^{(L_1+1)}, \cdots, w_{i,j}{}^{(L_2)}\right]^T \in \boldsymbol{C}^{L}, \tag{6}$$

where $w_{i,j}{}^{(k)}$ is the (i,j)th element of matrix $\boldsymbol{W}^{(k)}$.

Inouye and Tanebe [2] proposed the *multichannel super-exponential algorithm* for finding the tap coefficient vectors $\tilde{\boldsymbol{w}}_i$'s of the equalizer $\boldsymbol{W}(z)$, of which each iteration consists of the following two steps:

$$\tilde{\boldsymbol{w}}_i^{[1]} = \tilde{\boldsymbol{R}}_L^{\dagger} \tilde{\boldsymbol{d}}_i \qquad \textit{for } i = \overline{1,n}, \tag{7}$$

$$\tilde{\boldsymbol{w}}_i^{[2]} = \frac{\tilde{\boldsymbol{w}}_i^{[1]}}{\sqrt{\tilde{\boldsymbol{w}}_i^{[1]H} \tilde{\boldsymbol{R}}_L \tilde{\boldsymbol{w}}_i^{[1]}}} \quad \textit{for } i = \overline{1,n}, \tag{8}$$

where $(\cdot)^{[1]}$ and $(\cdot)^{[2]}$ stand respectively for the result of the first step and the result of the second steps. Let $\tilde{\boldsymbol{y}}(t)$ be the Lm-column vector consisting of the L consecutive inputs of the equalizer define by

$$\tilde{\boldsymbol{y}}(t) := \left[\bar{\boldsymbol{y}}_1(t)^T, \bar{\boldsymbol{y}}_2(t)^T, \cdots, \bar{\boldsymbol{y}}_m(t)^T\right]^T \in \boldsymbol{C}^{mL}, \tag{9}$$

$$\bar{\boldsymbol{y}}_i(t) := [y_i(t-L_1), y_i(t-L_1-1), \cdots, y_i(t-L_2)]^T \in \boldsymbol{C}^L, \tag{10}$$

where $y_i(t)$ is the ith element of the output vector $\boldsymbol{y}(t)$ of the channel system in (1). Then the correlation matrix $\tilde{\boldsymbol{R}}_L$ is represented as

$$\tilde{\boldsymbol{R}}_L = E\left[\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)\right] \in \boldsymbol{C}^{mL \times mL}, \tag{11}$$

and the forth-order cumulant vector $\tilde{\boldsymbol{d}}_i$ is represented as

$$\begin{aligned} \tilde{\boldsymbol{d}}_i \;=\; & E\left[|z_i(t)|^2 z_i(t) \tilde{\boldsymbol{y}}^*(t)\right] \\ & -2E\left[|z_i(t)|^2\right] E\left[z_i(t)\tilde{\boldsymbol{y}}^*(t)\right] \\ & -E\left[z_i{}^2(t)\right] E\left[z_i{}^*(t)\tilde{\boldsymbol{y}}^*(t)\right], \end{aligned} \tag{12}$$

where E$[x]$ denotes the expectation of a random variable x. We note that the last term can be ignored in case of E$[s_i{}^2(t)]$=0 for all $i = \overline{1,n}$, in which case E$[z_i{}^2(t)]$=0 for all $i = \overline{1,n}$.

3. PROPERTIES OF CORRELATION MATRICES AND MATRIX PSEUDO-INVERSION

We consider the rank deficiency problem of the correlation matrix $\tilde{\boldsymbol{R}}_L$ of $\{\tilde{\boldsymbol{y}}(t)\}$ in (11) with respect to the length $L{=}L_2{-}L_1{+}1$ of the equalizer $\boldsymbol{W}(z)$. This problem is very important for solving the equation (7) and also a fundamental issue in low-rank adaptive signal processing [6].

Theorem 1: Let $\tilde{\boldsymbol{R}}_L \in \boldsymbol{C}^{mL \times mL}$ be the correlation matrix defined by (11) for the channel system $\boldsymbol{H}(z)$ with n inputs and m outputs satisfying A1) and A2), where $L{=}L_2 - L_1 + 1$ is the length of the equalizer $\boldsymbol{W}(z)$. Then the following statements hold true:
1) If $m = n$, the $\tilde{\boldsymbol{R}}_L$ is nonsingular for $L = 1, 2, \cdots$.
2) If $m > n$ and $\boldsymbol{H}(z)$ is the transfer function of an FIR system of length K, then the sequence $\{\tilde{\boldsymbol{R}}_L\}$ decreases monotonically as L increases, and

$$\text{rank}\tilde{\boldsymbol{R}}_L = nL, \;\; \textit{for } L \geq nK. \tag{13}$$

3) If $m > n$ and if $\boldsymbol{H}(z)$ is the rational transfer function of an IIR system, then the sequence $\{\text{rank}\tilde{\boldsymbol{R}}_L\}$ decreases as L increases, and

$$\lim_{L\to\infty} \frac{1}{L}\text{rank}\tilde{\boldsymbol{R}}_L = n. \tag{14}$$

In order to develop an adaptive version of the multichannel super-exponential algorithm, the matrix inversion lemma [9] should be extended to the rank-degenerate case. The following lemma gives an explicit formula of the pseudoinverse for a positive semidefinite Hermitian matrix $\boldsymbol{A}$ added to a general rank-one matrix $\boldsymbol{b}\boldsymbol{b}^H$.

Lemma 1: Let $\boldsymbol{A} \in \boldsymbol{C}^{n\times n}$ be a positive semidefinite Hermitian matrix, and $\boldsymbol{b} \in \boldsymbol{C}^n$ be a nonzero vector. Let the linear vector space $\boldsymbol{C}^n$ be uniquely decomposed as $\boldsymbol{C}^n$=Im$\boldsymbol{A}\oplus$(Im$\boldsymbol{A})^\perp$, where Im$\boldsymbol{A}$ denotes the image space of $\boldsymbol{A}$ and (Im$\boldsymbol{A})^\perp$ denotes the orthogonal complement of Im$\boldsymbol{A}$. Let $\boldsymbol{b} \in \boldsymbol{C}^n$ be decomposed uniquely as

$$\boldsymbol{b} = \boldsymbol{b}_1 \oplus \boldsymbol{b}_2 \;\textit{with}\; \boldsymbol{b}_1 \in \textit{Im}\boldsymbol{A} \;\textit{and}\; \boldsymbol{b}_2 \in (\textit{Im}\boldsymbol{A})^\perp. \tag{15}$$

Let $\boldsymbol{Q}$ be defined as

$$\boldsymbol{Q} = \boldsymbol{A} + \boldsymbol{b}\boldsymbol{b}^H \in \boldsymbol{C}^{n\times n}. \tag{16}$$

Then the pseudoinverse $\boldsymbol{Q}^\dagger$ of matrix $\boldsymbol{Q}$ is explicitly expressed, depending on the values of vectors $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ and matrix $\boldsymbol{A}$, as follows:

1) If $\boldsymbol{b}_2 = 0$, then

$$\boldsymbol{Q}^\dagger = \boldsymbol{A}^\dagger - \frac{\boldsymbol{A}^\dagger \boldsymbol{b}_1 \boldsymbol{b}_1^H \boldsymbol{A}^\dagger}{1 + \boldsymbol{b}_1^H \boldsymbol{A}^\dagger \boldsymbol{b}_1}. \quad (17)$$

2) If $\boldsymbol{b}_2 \neq 0$ and $\boldsymbol{b}_1 = 0$, then

$$\boldsymbol{Q}^\dagger = \boldsymbol{A}^\dagger + \frac{\boldsymbol{b}_2 \boldsymbol{b}_2^H}{(\boldsymbol{b}_2^H \boldsymbol{b}_2)^2}. \quad (18)$$

3) Let l be a non-negative number defined by

$$l := |1 + \boldsymbol{b}_1^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_2|^2 - \boldsymbol{b}_1^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_1 \boldsymbol{b}_2^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_2, \quad (19)$$

where $\boldsymbol{Q}_b^\dagger$ is defined by

$$\boldsymbol{Q}_b^\dagger := \boldsymbol{A}^\dagger - \frac{\boldsymbol{A}^\dagger \boldsymbol{b}_1 \boldsymbol{b}_1^H \boldsymbol{A}^\dagger}{1 + \boldsymbol{b}_1^H \boldsymbol{A}^\dagger \boldsymbol{b}_1} + \frac{\boldsymbol{b}_2 \boldsymbol{b}_2^H}{(\boldsymbol{b}_2^H \boldsymbol{b}_2)^2}. \quad (20)$$

Then in the case when $\boldsymbol{b}_1 \neq 0$, $\boldsymbol{b}_2 \neq 0$ and $l \neq 0$,

$$\boldsymbol{Q}^\dagger = \boldsymbol{Q}_b^\dagger - \boldsymbol{Q}_b^\dagger \left[\boldsymbol{b}_1, \boldsymbol{b}_2\right] \boldsymbol{Q}_d \left[\boldsymbol{b}_1, \boldsymbol{b}_2\right]^H \boldsymbol{Q}_b^\dagger, \quad (21)$$

where

$$\boldsymbol{Q}_d := \frac{1}{l} \left[\begin{array}{c|c} -\boldsymbol{b}_2^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_2 & 1 + \boldsymbol{b}_1^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_2 \\ \hline 1 + \boldsymbol{b}_2^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_1 & -\boldsymbol{b}_1^H \boldsymbol{Q}_b^\dagger \boldsymbol{b}_1 \end{array} \right] \in \boldsymbol{C}^{2\times 2}. \quad (22)$$

Remark 2: We can show $0 < l \leq 1$ under the assumptions.

4. ADAPTIVE SUPER-EXPONENTIAL ALGORITHMS

Except for the case when the number of outputs equals the number of inputs, i.e., $m = n$, the correlation matrix $\tilde{\boldsymbol{R}}_L$ is not of full rank. Situations with the number of independent sources (or inputs) being strictly less than the number of sensors (or outputs) are often encountered in various applications such as digital communication, image processing and neural information processing. Moreover, if the underlying channel system exhibits slow changes in time, processing all the available data jointly is not desirable, even if we can accommodate the computational and storage loads of the batch algorithm in (7) and (8), because different data segments correspond to different channel responses. In such a case, we want to have an adaptive algorithm which is capable of tracking the varying characteristics of the channel system. In the following, we propose two types of AMSEA's, that is, the one in covariance (correlation or Kalman-filter) form and the other in QR-factorization form.

Consider the batch algorithm in (7) and (8). The equation (8) constraints the length of vector $\tilde{\boldsymbol{w}}_i$ to equal one, and thus we assume this constraint is always satisfied using a normalization or an automatic gain control (AGC) of $\tilde{\boldsymbol{w}}_i$ at each time t. To develop an adaptive version of (7), we must specify the dependency of each time t and rewrite (7) as

$$\tilde{\boldsymbol{w}}_i(t) = \tilde{\boldsymbol{R}}_L^\dagger(t) \tilde{\boldsymbol{d}}_i(t) \quad , \quad i = \overline{1, n}. \quad (23)$$

Here the subscript L of $\tilde{\boldsymbol{R}}_L(t)$ is omitted for simplicity hereafter. The recursions for time-updating of matrix $\tilde{\boldsymbol{R}}(t)$ and vector $\tilde{\boldsymbol{d}}_i(t)$ in (23) are given as

$$\tilde{\boldsymbol{R}}(t) = \alpha \tilde{\boldsymbol{R}}(t-1) + (1-\alpha) \tilde{\boldsymbol{y}}^*(t) \tilde{\boldsymbol{y}}^T(t), \quad (24)$$

$$\tilde{\boldsymbol{d}}_i(t) = \alpha \tilde{\boldsymbol{d}}_i(t-1) + (1-\alpha) \tilde{\boldsymbol{y}}^*(t) \tilde{z}_i(t), \quad (25)$$

where

$$\tilde{z}_i(t) := (|z_i(t)|^2 - 2 < |z_i(t)|^2 >) z_i(t) - < z_i^2(t) > z_i^*(t). \quad (26)$$

Here $< |z_i(t)|^2 >$ and $< z_i^2(t) >$ denote respectively the estimates of $E\left[|z_i(t)|^2\right]$ and $E\left[z_i(t)^2\right]$ at time t, α is a positive constant close to, but less than one, which accounts for some exponential weighting factor or forgetting factor [9].

By applying Lemma 1 for calculating the pseudoinverse of $\tilde{\boldsymbol{R}}(t)$, we obtain the following theorem which determines $\tilde{\boldsymbol{w}}_i(t)$ from $\tilde{\boldsymbol{w}}_i(t-1)$, $\tilde{\boldsymbol{y}}(t)$ and $z_i(t)$.

Theorem 2: The recursion for $\tilde{\boldsymbol{w}}_i(t)$ is

$$\tilde{\boldsymbol{w}}_i(t) = \boldsymbol{P}(t) \tilde{\boldsymbol{R}}(t) \tilde{\boldsymbol{w}}_i(t-1) + \boldsymbol{k}(t) \left[\tilde{z}_i(t) - \tilde{\boldsymbol{y}}^T(t) \tilde{\boldsymbol{w}}_i(t-1)\right], \quad (27)$$

where

$$\boldsymbol{k}(t) := (1-\alpha) \boldsymbol{P}(t) \tilde{\boldsymbol{y}}^*(t), \quad (28)$$

$$\tilde{z}_i(t) := (|z_i(t)|^2 - 2 < |z_i(t)|^2 >) z_i(t) - < z_i^2(t) > z_i^*(t), \quad (29)$$

$$< |z_i(t)|^2 > := \beta < |z_i(t-1)|^2 > + (1-\beta) |z_i(t)|^2, \quad (30)$$

$$< z_i^2(t) > := \beta < z_i^2(t-1) > + (1-\beta) z_i^2(t), \quad (31)$$

and the formula of the recursion for $\boldsymbol{P}(t)$ from $\boldsymbol{P}(t-1)$ by using Lemma 1 is very lengthy and is omitted for page limit. Here β is a positive constant less than α. These equations are initialized by their values appropriately selected or calculated by the batch algorithm in (7) and (8) at initial time t_0 and used for $t = t_0 + 1$, $t_0 + 2, \cdots$.

Before presenting another type of adaptive algorithms, we mention the following lemma on the so-called QR-factorization of a general matrix $\boldsymbol{A}$.

Lemma 2 [8],[10]**:** Given an $n \times n$ Hermitian $\boldsymbol{A} \in \boldsymbol{C}^{n\times n}$. Let r be a chosen integer satisfying $|\lambda_r| > |\lambda_{r+1}|$, where the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $\boldsymbol{A}$ are arranged in decreasing order of magnitude. Given an $n \times r$ matrix $\boldsymbol{Q_0}$ with orthonormal columns and generate a sequence of matrices $\{\boldsymbol{Q_k}\} \subset \boldsymbol{C}^{n\times r}$ as follows:

$$\boldsymbol{Z}_k = \boldsymbol{A} \boldsymbol{Q}_{k-1}, \quad (32)$$

$$\boldsymbol{Q}_k \boldsymbol{R}_k = \boldsymbol{Z}_k \quad : \textit{QR-factorization}, \quad (33)$$

where $\boldsymbol{Q}_k \in \boldsymbol{C}^{n\times r}$ is a matrix with orthonormal columns and $\boldsymbol{R}_k \in \boldsymbol{C}^{r\times r}$ is an upper triangular matrix. If $\boldsymbol{Q_0}$ is not unfortunately chosen, then the sequence $\{\boldsymbol{Q}_k\}$ converges to a matrix of r dominant eigenvectors, and the upper triangular sequence $\{\boldsymbol{R}_k\}$ converges the diagonal matrix of r dominant eigenvalues.

By applying Lemma 2 for calculating the pseudoinverse of $\tilde{\boldsymbol{R}}(t)$, we have the following theorem which gives an adaptive solution $\tilde{\boldsymbol{w}}_i(t)$ of (23) from $\boldsymbol{Q}_r(t-1)$, $\boldsymbol{Q}_r(t-2)$, $\tilde{\boldsymbol{d}}_i(t-1)$, $\tilde{\boldsymbol{y}}(t)$ and $z_i(t)$ (where, for example, $\boldsymbol{Q}_r(t-1) \in \boldsymbol{C}^{mL\times r}$ represents approximately r dominants eigenvectors of $mL \times mL$ matrix $\tilde{\boldsymbol{R}}(t-1)$).

Theorem 3: Let r be fixed as $r = nL$, where n is the number of the inputs of the channel system in (1) and L is the length of the equalizer in (4). Then an adaptive solution $\tilde{\boldsymbol{w}}_i(t)$ of (23) is

$$\tilde{\boldsymbol{w}}_i(t) = \boldsymbol{Q}_r(t-1)\boldsymbol{R}_r^{-1}(t)\boldsymbol{Q}_r^H(t)\tilde{\boldsymbol{d}}_i(t), \tag{34}$$

where $\boldsymbol{Q}_r(t)$ and $\boldsymbol{R}_r(t)$ is obtained by the QR decomposition of matrix $\boldsymbol{Z}(t)$ defined by $\boldsymbol{Z}(t) := \tilde{\boldsymbol{R}}(t)\,\boldsymbol{Q}_r(t-1)$, which is decomposed as

$$\begin{aligned} \boldsymbol{Z}(t) &= \boldsymbol{Q}_r(t)\boldsymbol{R}_r(t) \in \boldsymbol{C}^{mL\times r}, \\ & \boldsymbol{Q}_r(t) \in \boldsymbol{C}^{mL\times r}, \quad \boldsymbol{R}_r(t) \in \boldsymbol{C}^{r\times r}, \end{aligned} \tag{35}$$

and the update of $\boldsymbol{Z}(t)$ is

$$\begin{aligned} \boldsymbol{Z}(t) &= \alpha\boldsymbol{Z}(t-1)\boldsymbol{Q}_r^H(t-2)\boldsymbol{Q}_r(t-1) \\ &+ (1-\alpha)\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)\boldsymbol{Q}_r(t-1). \end{aligned} \tag{36}$$

The update of $\tilde{\boldsymbol{d}}_i(t)$ is

$$\tilde{\boldsymbol{d}}_i(t) = \alpha\tilde{\boldsymbol{d}}_i(t-1) + (1-\alpha)\tilde{\boldsymbol{y}}^*(t)\tilde{z}_i(t), \tag{37}$$

where

$$\tilde{z}_i(t) := (|z_i(t)|^2 - 2 < |z_i(t)|^2 >)z_i(t) - < z_i^2(t) > z_i^*(t), \tag{38}$$

$$< |z_i(t)|^2 >= \beta < |z_i(t-1)|^2 > + (1-\beta)|z_i(t)|^2, \tag{39}$$

$$< z_i^2(t) >= \beta < z_i^2(t-1) > + (1-\beta)z_i^2(t). \tag{40}$$

These equations are initialized by their values appropriately selected or calculated by the batch algorithm in (7) and (8) at an initial time t_0 and used for $t = t_0 + 1$, $t_0 + 2, \cdots$.

Remark 3: If the number n of inputs varies dynamically, we should estimate the number of the inputs before using Theorem 3.

5. SIMULATIONS

For page limit, we show only one of the simulation results in Figure 1 by using the AMSEA in covariance form (23) - (31). We considered an MIMO channel system with two inputs and three outputs, and assumed that the length of the channel is three ($K = 3$), the length of the equalizer is six ($L = 6$), and two source signals are the 4-PSK and the 8-PSK signals. As a measure of performance, we use the multichannel intersymbol interference (MISI). The last matrix $\boldsymbol{H}^{(2)}$ of the impulse response of the channel was varied by approximately 3 times at discrete time t=10,000. The values of α and β were chosen as α=0.999 and β=0.05, respectively. Figure 1 shows the result for the time-variant system obtained by using 50,000 data samples. The details of the simulation results will be shown in the conference.

6. CONCLUSIONS

We have investigated multichannel blind deconvolution of FIR or IIR systems using the multichannel super-exponential method. We have shown some properties of the correlation matrices relevant to the multichannel super-exponential method and then presented a pseudo-inversion lemma. Based on these results, we have proposed two types of adaptive multichannel super-exponential algorithms (AMSEA's), the one in covariance form and the other in QR-factorization form.

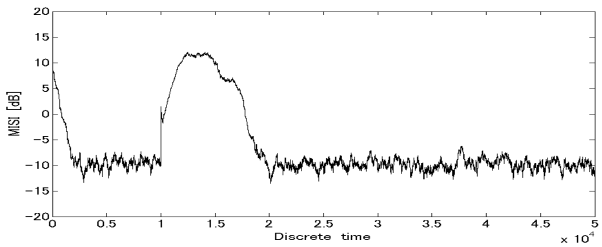

Figure 1: Performance of AMSEA in covariance form.

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