A Modified Eigenvector Method for Blind Deconvolution of MIMO Systems Using the Matrix Pseudo-Inversion Lemma

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***Abstract*— Recently we have developed an eigenvector method (EVM) which can achieve the blind deconvolution (BD) for MIMO systems. The attractive features of the proposed algorithm are that the BD can be achieved by calculating the eigenvectors of a matrix and by using reference signals. However, the performance accuracy of the EVM depends highly on the computational result of the eigenvectors. In this paper, by modifying the EVM, we propose an algorithm which can achieve the BD without calculating the eigenvectors. Then the pseudo-inverse which is needed to carry out the BD is calculated by our proposed matrix pseudo-inversion lemma. Simulation results will be presented for showing the validity of the proposed method.**

I. Introduction

In this paper, we deal with a blind deconvolution (BD) problem for a multiple-input and multiple-output (MIMO) infinite-impulse response (IIR) channels. A large number of methods for solving the BD problem have been proposed until now (see [1], and reference therein). In order to solve the BD problem, this paper focuses on the eigenvector method (EVM).

The first proposal of the EVM was done by Jelonnek et al. [4]. They have proposed the EVM for solving blind equalization (BE) problems of single-input single-output (SISO) channels and single-input multiple-output (SIMO) channels. The most attractive feature of the EVM is that its algorithm can be derived from a closed-form solution using reference signals. Then, a generalized eigenvector problem can be formulated and the eigenvector calculation is carried out in order to solve the BE problem. Owing to the property, differently from the algorithms derived from steepest descent methods, the EVM does not need many iterations to achieve the BE, but works so as to solve the BE problem with one iteration. Recently, we extended the EVM to the case of MIMO-IIR channels [6]. Then we proved that the proposed EVM can work so as to recover all source signals from their mixtures with one iteration. However, in the EVM, its performance accuracy depends highly on the computational result of the eigenvectors.

In this paper, we modify the EVM and then an algorithm for solving the BD is proposed, in which the proposed algorithm can be carried out without calculating the eigenvectors. Namely, the proposed algorithm can achieve the BD with as less computational complexity as possible, compared with the conventional EVMs.

The present paper uses the following notation: Let Z denote the set of all integers. Let C denote the set of all complex numbers. Let $\boldsymbol{C}^n$ denote the set of all n-column vectors with complex components. Let $\boldsymbol{C}^{m\times n}$ denote the set of all $m \times n$ matrices with complex components. The superscripts T, $*$, and H denote, respectively, the transpose, the complex conjugate, and the complex conjugate transpose (Hermitian) of a matrix. The symbols block-diag$\{\cdots\}$ and diag$\{\cdots\}$ denote respectively a block diagonal and a diagonal matrices with the block diagonal and the diagonal elements $\{\cdots\}$. The symbol cum$\{x_1,x_2,x_3,x_4\}$ denotes the fourth-order cumulant of x_i's. Let $i = \overline{1,n}$ stand for $i = 1, 2, \cdots, n$.

II. Problem Formulation and Assumptions

We consider a MIMO system with n inputs and m outputs as described by

$$\boldsymbol{y}(t) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k) + \boldsymbol{n}(t), \quad t \in Z, \qquad (1)$$

where $\boldsymbol{s}(t)$ is an n-column vector of input (or source) signals, $\boldsymbol{y}(t)$ is an m-column vector of system outputs, $\boldsymbol{n}(t)$ is an m-column vector of Gaussian noises, and $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ impulse response matrix sequence. The transfer function of the system is defined by $\boldsymbol{H}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} z^k, z \in C$.

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To recover the source signals, we process the output signals by an $n \times m$ deconvolver (or equalizer) $\boldsymbol{W}(z)$ described by

$$\begin{aligned}\boldsymbol{v}(t) &= \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{y}(t-k) \\ &= \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} \boldsymbol{s}(t-k) + \sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{n}(t-k), \quad (2)\end{aligned}$$

where $\{\boldsymbol{G}^{(k)}\}$ is the impulse response matrix sequence of $\boldsymbol{G}(z) := \boldsymbol{W}(z)\boldsymbol{H}(z)$, which is defined by $\boldsymbol{G}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} z^k, z \in C$. The cascade connection of the unknown system and the deconvolver is illustrated in Fig. 1.

Here, we put the following assumptions on the system, the source signals, the deconvolver, and the noises.

A1) The transfer function $\boldsymbol{H}(z)$ is stable and has full column rank on the unit circle $|z| = 1$, where the assumption **A1)** implies that the unknown system has less inputs than outputs, i.e., $n \leq m$, and there exists a left stable inverse of the unknown system.

A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a complex, zero-mean and non-Gaussian random vector process with element processes $\{s_i(t)\}$, $i = \overline{1,n}$ being mutually independent. Each element process $\{s_i(t)\}$ is an i.i.d. process with a variance $\sigma_{s_i}^2 \neq 0$ and a nonzero fourth-order cumulant $\gamma_i \neq 0$ defined as

$$\gamma_i = \mathrm{cum}\{s_i(t), s_i(t), s_i^*(t), s_i^*(t)\} \neq 0. \quad (3)$$

A3) The deconvolver $\boldsymbol{W}(z)$ is an FIR system, that is, $\boldsymbol{W}(z) = \sum_{L_1}^{L_2} \boldsymbol{W}^{(k)} z^k$, where the length $L := L_2 - L_1 + 1$ is taken to be sufficiently large so that the truncation effect can be ignored.

A4) The noise sequence $\{\boldsymbol{n}(t)\}$ is a zero-mean, Gaussian vector stationary process whose component processes $\{n_j(t)\}$, $j = \overline{1,m}$ have nonzero variances $\sigma_{n_j}^2$, $j = \overline{1,m}$.

A5) The two vector sequences $\{\boldsymbol{n}(t)\}$ and $\{\boldsymbol{s}(t)\}$ are mutually statistically independent.

Under **A3)**, the impulse response $\{\boldsymbol{G}^{(k)}\}$ of the cascade system is given by

$$\boldsymbol{G}^{(k)} := \textstyle\sum_{\tau=L_1}^{L_2} \boldsymbol{W}^{(\tau)} \boldsymbol{H}^{(k-\tau)}, \quad k \in Z, \quad (4)$$

In a vector form, (4) can be written as

$$\tilde{\boldsymbol{g}}_i = \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i, \quad i = \overline{1,n}, \quad (5)$$

where $\tilde{\boldsymbol{g}}_i$ is the column vector consisting of the ith output impulse response of the cascade system defined by $\tilde{\boldsymbol{g}}_i := [\boldsymbol{g}_{i1}^T, \boldsymbol{g}_{i2}^T, \cdots, \boldsymbol{g}_{in}^T]^T$,

$$\boldsymbol{g}_{ij} := [\cdots, g_{ij}(-1), g_{ij}(0), g_{ij}(1), \cdots]^T, \quad j = \overline{1,n} \quad (6)$$

where $g_{ij}(k)$ is the (i,j)th element of matrix $\boldsymbol{G}^{(k)}$, and $\tilde{\boldsymbol{w}}_i$ is the mL-column vector consisting of the tap coefficients (corresponding to the ith output) of the deconvolver defined by $\tilde{\boldsymbol{w}}_i := \left[\boldsymbol{w}_{i1}^T, \boldsymbol{w}_{i2}^T, \cdots, \boldsymbol{w}_{im}^T\right]^T \in \boldsymbol{C}^{mL}$,

$$\boldsymbol{w}_{ij} := [w_{ij}(L_1), w_{ij}(L_1+1), \cdots, w_{ij}(L_2)]^T \in \boldsymbol{C}^L, \quad (7)$$

$j = \overline{1,m}$, where $w_{ij}(k)$ is the (i,j)th element of matrix $\boldsymbol{W}^{(k)}$, and $\tilde{\boldsymbol{H}}$ is the $n \times m$ block matrix whose (i,j)th block element $\boldsymbol{H}_{ij}$ is the matrix (of L columns and possibly infinite number of rows) with the (l,r)th element $[\boldsymbol{H}_{ij}]_{lr}$ defined by $[\boldsymbol{H}_{ij}]_{lr} := h_{ji}(l-r)$, $l = 0, \pm 1, \pm 2, \cdots$, $r = \overline{L_1, L_2}$, where $h_{ij}(k)$ is the (i,j)th element of the matrix $\boldsymbol{H}^{(k)}$.

In the MIMO deconvolution problem, we want to adjust $\tilde{\boldsymbol{w}}_i$'s (i = $\overline{1,n}$) so that

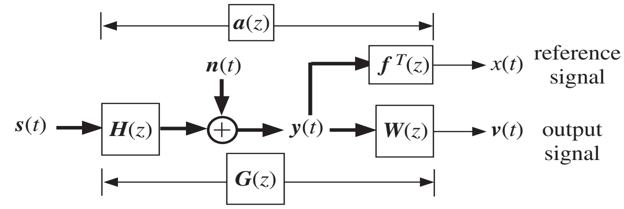

Fig. 1. The composite system of an unknown system and a deconvolver, and a reference system (The case of single reference).

$$[\tilde{\boldsymbol{g}}_1, \cdots, \tilde{\boldsymbol{g}}_n] = \tilde{\boldsymbol{H}}[\tilde{\boldsymbol{w}}_1, \cdots, \tilde{\boldsymbol{w}}_n] = [\tilde{\boldsymbol{\delta}}_1, \cdots, \tilde{\boldsymbol{\delta}}_n]\boldsymbol{P}, \quad (8)$$

where $\boldsymbol{P}$ is an $n \times n$ permutation matrix, and $\tilde{\boldsymbol{\delta}}_i$ is the n-block column vector defined by $\tilde{\boldsymbol{\delta}}_i := [\boldsymbol{\delta}_{i1}^T, \boldsymbol{\delta}_{i2}^T, \ldots, \boldsymbol{\delta}_{in}^T]^T$, $i = \overline{1,n}$, $\boldsymbol{\delta}_{ij} := \hat{\boldsymbol{\delta}}_i$, for $i = j$, otherwise $(\cdots, 0, 0, 0, \cdots)^T$. Here, $\hat{\boldsymbol{\delta}}_i$ is the column vector (of infinite elements) whose rth element $\hat{\delta}_i(r)$ is given by $\hat{\delta}_i(r) = d_i \delta(r - k_i)$, where $\delta(t)$ is the Kronecker delta function, d_i is a complex number standing for a scale change and a phase shift, and k_i is an integer standing for a time shift.

III. The Conventional Eigenvector Algorithm

Jelonnek et al. [4] have shown in the single-input case that from the following problem, that is,

$$\begin{aligned}&\text{Maximize } D_{v_i x} = \mathrm{cum}\{v_i(t), v_i^*(t), x(t), x^*(t)\} \\ &\text{under } \sigma_{v_i}^2 = \sigma_{s_{\rho_i}}^2, \quad (9)\end{aligned}$$

a closed-form solution expressed as a generalized eigenvector problem can be led by the Lagrangian method, where $\sigma_{v_i}^2$ and $\sigma_{s_{\rho_i}}^2$ denote the variances of the output $v_i(t)$ and a source signal $s_{\rho_i}(t)$, respectively, ρ_i is one of integers $\{1, 2, \cdots, n\}$ such that the set $\{\rho_1, \rho_2, \cdots, \rho_n\}$ is a permutation of the set $\{1, 2, \cdots, n\}$, $v_i(t)$ is the ith element of $\boldsymbol{v}(t)$ in (2), and the reference signal $x(t)$ is given by $\boldsymbol{f}^T(z)\boldsymbol{y}(t)$ using an appropriate filter $\boldsymbol{f}(z)$ (see Fig. 1). The filter $\boldsymbol{f}(z)$ is called a *reference system*. Let $\boldsymbol{a}(z) := \boldsymbol{H}^T(z)\boldsymbol{f}(z) = [a_1(z), a_2(z), \cdots, a_n(z)]^T$, then $x(t) = \boldsymbol{f}^T(z)\boldsymbol{H}(z)\boldsymbol{s}(t) = \boldsymbol{a}^T(z)\boldsymbol{s}(t)$. The element $a_i(z)$ of the filter $\boldsymbol{a}(z)$ is defined as $a_i(z) = \sum_{k=-\infty}^{\infty} a_i(k) z^k$ and the reference system $\boldsymbol{f}(z)$ is an m-column vector whose elements are $f_j(z) = \sum_{k=L_1}^{L_2} f_j(k) z^k$, $j = \overline{1,m}$.

In our case, $D_{v_i x}$ and $\sigma_{v_i}^2$ can be expressed in terms of the vector $\tilde{\boldsymbol{w}}_i$ as, respectively, $D_{v_i x} = \tilde{\boldsymbol{w}}_i^H \tilde{\boldsymbol{B}} \tilde{\boldsymbol{w}}_i$ and $\sigma_{v_i}^2 = \tilde{\boldsymbol{w}}_i^H \tilde{\boldsymbol{R}} \tilde{\boldsymbol{w}}_i$, where $\tilde{\boldsymbol{B}}$ is the $m \times m$ block matrix whose (i,j)th block element $\boldsymbol{B}_{ij}$ is the matrix with the (l,r)th element $[\boldsymbol{B}_{ij}]_{lr}$ calculated by $\mathrm{cum}\{y_i^*(t\text{-}L_1\text{-}l\text{+}1), y_j(t\text{-}L_1\text{-}r\text{+}1), x^*(t), x(t)\}$ $(l, r = \overline{1,L})$ and $\tilde{\boldsymbol{R}} = E[\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)]$ is the covariance matrix of m-block column vector $\tilde{\boldsymbol{y}}(t)$ defined by

$$\tilde{\boldsymbol{y}}(t) := \left[\boldsymbol{y}_1^T(t), \boldsymbol{y}_2^T(t), \cdots, \boldsymbol{y}_m^T(t)\right]^T \in \boldsymbol{C}^{mL}, \quad (10)$$

where $\boldsymbol{y}_j(t) := [y_j(t\text{-}L_1), y_j(t\text{-}L_1\text{-}1), \cdots, y_j(t\text{-}L_2)]^T \in \boldsymbol{C}^L$, $j = \overline{1,m}$. It follows from (10) that $\tilde{\boldsymbol{y}}(t)$ is expressed as $\tilde{\boldsymbol{y}}(t) = \boldsymbol{D}_c(z)\boldsymbol{y}(t)$, where $\boldsymbol{D}_c(z)$ is an $mL \times m$ converter (consisting of m identical delay chains each of which has L delay elements when $L_1 = 1$) defined by $\boldsymbol{D}_c(z) :=$ block-diag$\{\boldsymbol{d}_c(z), \cdots, \boldsymbol{d}_c(z)\}$ with m diagonal block elements all being the same L-column vector $\boldsymbol{d}_c(z)$ defined by $\boldsymbol{d}_c(z) =$

$[z^{L_1},\cdots,z^{L_2}]^T$. Therefore, by the similar way to as in [4], the maximization of $|D_{v_i x}|$ under $\sigma^2_{v_i} = \sigma^2_{s_{\rho_i}}$ leads to the following generalized eigenvector problem;

$$\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i. \tag{11}$$

Moreover, Jelonnek et al. have shown in [4] that the eigenvector corresponding to the maximum magnitude eigenvalue of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}$ becomes the solution of the blind equalization problem, which is referred to as an *eigenvector algorithm* (EVA). It has been also shown in [6] that the BD for MIMO-IIR systems can be achieved with the eigenvectors of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}$, using only one reference signal. Note that since Jelonnek et al. have dealt with SISO-IIR systems or SIMO-IIR systems, the constructions of $\tilde{\boldsymbol{B}}$, $\tilde{\boldsymbol{w}}_i$, and $\tilde{\boldsymbol{R}}$ in (11) are different from those proposed in [4].

Castella et al. [2] have shown that from (9), a BD can be iteratively achieved by using $x_i(t) = \tilde{\boldsymbol{w}}_i\tilde{\boldsymbol{y}}(t)$ $(i = \overline{1,n})$ as reference signals (see Fig. 2), where the number of reference signals corresponds to the number of source signals and $\tilde{\boldsymbol{w}}_i$ is an eigenvector obtained by $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i$ in the previous iteration, where $\tilde{\boldsymbol{B}}_i$ represents $\tilde{\boldsymbol{B}}$ in (11) calculated by $x_i(t) = \tilde{\boldsymbol{w}}_i\tilde{\boldsymbol{y}}(t)$. Namely, they considered the following equation;

$$\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{w}}_i. \tag{12}$$

Then a deflation method was used to recover all source signals. However, the EVM proposed by Castella et al. requires the calculation of the eigenvectors of the matrix $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i$ to achieve the BD.

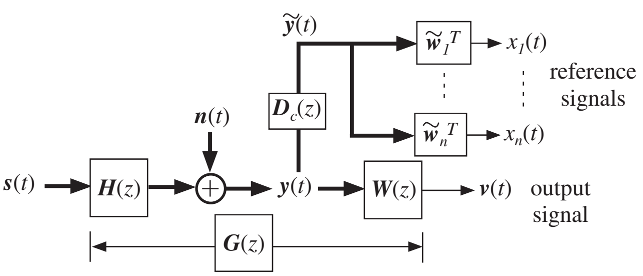

Fig. 2. The composite system of an unknown system and a deconvolver, and a reference system (The case of multiple reference system).

IV. The proposed algorithm

Here, the equation (12) can be interpreted as follows. Suppose that the value $\tilde{\boldsymbol{w}}_i$ in the left-hand side of (12) is an eigenvector obtained by $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i$ in the previous iteration. Also, let $\tilde{\boldsymbol{d}}_i$ denote $\tilde{\boldsymbol{B}}_i\tilde{\boldsymbol{w}}_i$. Then (12) can be expressed as

$$\tilde{\boldsymbol{w}}_i = \frac{1}{\lambda_i}\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{d}}_i, \quad i = \overline{1,n}. \tag{13}$$

Differently from the EVM in [2], (13) means that $\tilde{\boldsymbol{w}}_i$ is modified iteratively by the value of the right-hand side of (13) without calculating the eigenvectors of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i$, where $\tilde{\boldsymbol{w}}_i$ in both $x_i(t)$ and $\tilde{\boldsymbol{d}}_i$ is the value of the left-hand side of (13) in the previous iteration. The scalar λ_i is fixed to be 1, but $\tilde{\boldsymbol{w}}_i$ obtained by (13) should be normalized at each iteration, that is,

$$\tilde{\boldsymbol{w}}_i := \frac{\tilde{\boldsymbol{w}}_i}{\sqrt{\tilde{\boldsymbol{w}}_i^H\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i}}, \quad i = \overline{1,n}. \tag{14}$$

It can be seen that the iterative algorithm (13) is nothing but an iterative procedure of the SEM [7]. Therefore, our proposed algorithm for achieving the BD is that the vector $\tilde{\boldsymbol{w}}_i$ is modified by using the value $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{d}}_i$ in (13), and then the modified vector, that is, $\tilde{\boldsymbol{w}}_i$ in the left-hand side of (13) is normalized by (14).

Here, the calculation of $\tilde{\boldsymbol{R}}^{\dagger}$ is implemented by using the following algorithm based on the matrix pseudo-inversion lemma proposed in [9]. The reason is that in the case that the pseudo-inverse is calculated using data block, the convergence speed is increased and the computational complexity is reduced, compared with the conventional pseudo-inverse algorithms, for example, the built-in function "pinv" in MATLAB [10]. Therefore, in order to provide a recursive formula based on block data for time-updating of pseudo-inverse, the block index k is defined, and then $\tilde{\boldsymbol{R}}$ and $\tilde{\boldsymbol{R}}^{\dagger}$ are described as $\tilde{\boldsymbol{R}}(k)$ and $\boldsymbol{P}(k)$, respectively, where the k-th block of data is defined as

$$t = kl + i, \quad i = \overline{1, l-1}, \quad k \in \boldsymbol{Z}, \tag{15}$$

the parameters l and t denote the block length and the original discrete (or sample) time. The matrix $\tilde{\boldsymbol{R}}(k)$ is obtained by

$$\tilde{\boldsymbol{R}}(k) = (1-\alpha_k)\tilde{\boldsymbol{R}}(k-1) + \alpha_k \boldsymbol{Y}^*(k)\boldsymbol{Y}^T(k), \tag{16}$$

where

$$\begin{aligned}\boldsymbol{Y}(k) \;&=\; [\tilde{\boldsymbol{y}}\{(k-1)l\}, \tilde{\boldsymbol{y}}\{(k-1)l+1\}, \cdots,\\ &\qquad \tilde{\boldsymbol{y}}\{(k-1)l+l-1\}] \;\in\; \boldsymbol{C}^{mL\times l},\end{aligned} \tag{17}$$

and α_t is a positive number close to, but greater than zero, which accounts for some exponential weighting factor or forgetting factor [3]. Moreover, the following parameters are defined;

$$\boldsymbol{Y}(k) = \sqrt{\alpha_t}\tilde{\boldsymbol{Y}}^*(k), \tag{18}$$

$$\boldsymbol{Y}_1(k) = \tilde{\boldsymbol{R}}(k-1)\boldsymbol{P}(k-1)\boldsymbol{Y}(k), \tag{19}$$

$$\boldsymbol{Y}_2(k) = \{\boldsymbol{I} - \tilde{\boldsymbol{R}}(k-1)\boldsymbol{P}(k-1)\}\boldsymbol{Y}(k). \tag{20}$$

Then the pseudo-inverse $\boldsymbol{P}(k)$ can be explicitly expressed, as follows:

$$\begin{aligned}\boldsymbol{P}(k) = \boldsymbol{P}_B^{\dagger}(k) - \boldsymbol{P}_B^{\dagger}(k)\,[\boldsymbol{Y}_1(k), \boldsymbol{Y}_2(k)]\\ \boldsymbol{P}_D^{-1}(k)\,[\boldsymbol{Y}_1(k), \boldsymbol{Y}_2(k)]^H\,\boldsymbol{P}_B^{\dagger}(k),\end{aligned} \tag{21}$$

where $\boldsymbol{P}_B^{\dagger}(k)$ and $\boldsymbol{P}_D^{-1}(k)$ are respectively defined by

$$\begin{aligned}\boldsymbol{P}_B^{\dagger}(k) := \frac{1}{1-\alpha_k}[\boldsymbol{P}(k-1) - \boldsymbol{P}(k-1)\boldsymbol{Y}_1(k)\boldsymbol{P}_A^{-1}(k)\\ \boldsymbol{Y}_1^H(k)\boldsymbol{P}(k-1)] + \left(\boldsymbol{Y}_2^H(k)\right)^{\dagger}\boldsymbol{Y}_2^{\dagger}(k),\end{aligned} \tag{22}$$

and

$$\boldsymbol{P}_D^{-1}(k) := \left[\begin{array}{c|c} -\boldsymbol{\Delta}^{-1}(k)\boldsymbol{P}_2(k) & \boldsymbol{\Delta}^{-1}(k) \\ \hline \boldsymbol{I} + \boldsymbol{E}_1(k)\boldsymbol{\Delta}^{-1}(k)\boldsymbol{E}_2(k) & -\boldsymbol{E}_1(k)\boldsymbol{\Delta}^{-1}(k)\end{array}\right] \tag{23}$$

with

$$\boldsymbol{\Delta}(k) := \boldsymbol{I} - \boldsymbol{E}_2(k)\boldsymbol{E}_1(k), \tag{24}$$

where

$$\boldsymbol{E}_1(k) = \boldsymbol{B}_1^H(k)\boldsymbol{P}_B^\dagger(k)\boldsymbol{B}_1(k), \tag{25}$$

$$\boldsymbol{E}_2(k) = \boldsymbol{B}_2^H(k)\boldsymbol{P}_B^\dagger(k)\boldsymbol{B}_2(k). \tag{26}$$

We treat $\boldsymbol{P}(k)$ as $\tilde{\boldsymbol{R}}^\dagger$, and $\tilde{\boldsymbol{w}}_i$ is iteratively modified using (13) and (14), where λ_i in (13) is assumed to be fixed to 1 and $\tilde{\boldsymbol{d}}_i := \tilde{\boldsymbol{B}}_i\tilde{\boldsymbol{w}}_i$ in (13) is estimated by using $\boldsymbol{Y}(k)$.

V. Simulation Results

To demonstrate the proposed algorithm, we considered a MIMO system $\boldsymbol{H}(z)$ with two inputs ($n = 2$) and three outputs ($m = 3$), and assumed that the system $\boldsymbol{H}(z)$ is FIR and the length of channel is three, that is $\boldsymbol{H}^{(k)}$'s in (1) were set to be

$$\boldsymbol{H}(z) = \sum_{k=0}^{2}\boldsymbol{H}^{(k)}z^k = \begin{bmatrix} 1.00 + 0.15z + 0.10z^2 & 0.65 + 0.25z + 0.15z^2 \\ 0.50 - 0.10z + 0.20z^2 & 1.00 + 0.25z + 0.10z^2 \\ 0.60 + 0.10z + 0.40z^2 & 0.10 + 0.20z + 0.10z^2 \end{bmatrix}. \tag{27}$$

The source signals $s_1(t)$ and $s_2(t)$ were a sub-Gaussian signal which takes one of two values, -1 and 1 with equal probability 1/2. The parameters L_1 and L_2 in $\boldsymbol{W}(z)$ were set to be 0 and 9, respectively. As a measure of performances, we used the *multichannel intersymbol interference* (M_{ISI}) [8], which was the average of 50 Monte Carlo runs. In each Monte Carlo run, using 300 data samples, $\tilde{\boldsymbol{w}}_i$ is modified by (13) and (14), and the total number of modification times is 10. The block length l is set to be 2. For obtaining the pseudo-inverse of the correlation matrix, the initial values of $\tilde{\boldsymbol{R}}$, $\tilde{\boldsymbol{d}}_i$ and $\boldsymbol{P}$ were estimated using 30 data samples. The value of α_k was chosen as $\alpha_k = \frac{1}{kl}$ for each k.

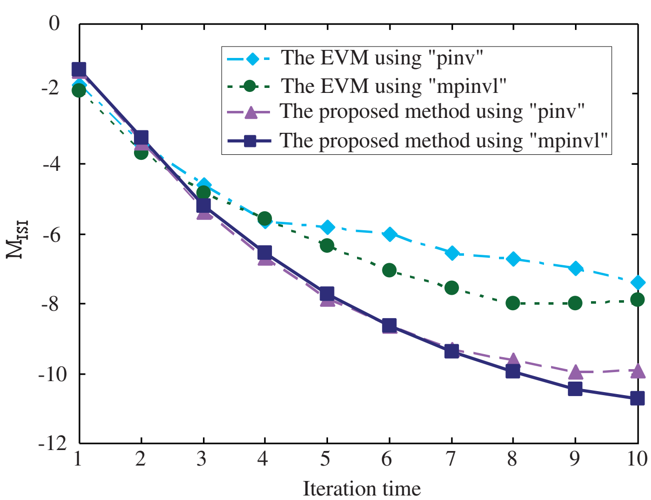

Fig. 3. The performances of the proposed algorithm and the conventional methods.

Figure 3 shows the results obtained by the proposed algorithm and the conventional methods. As the conventional method, we selected the EVM proposed by Castella et al.. Then, the pseudo-inverse of $\boldsymbol{R}$ in (12) was calculated by the built-in function "pinv" in MATLAB and our proposed matrix pseudo-inversion lemma, denoted by "mpinvl". From Fig. 3, one can see that the performance of the proposed algorithm is better than that of the conventional EVMs.

Table 1 shows the average of the execution times for the proposed method and the conventional EVM, using a personal computer (Windows machine) with 2.59GHz processor and 2GB main memories. From the table 1, one can see that the execution time of the proposed method is longer than that of the conventional EVM. However, since the difference is very small and the performance of the conventional EVM is worse than the proposed algorithm, then we conclude that the proposed algorithm is more suitable for solving the BD problem than the conventional EVM.

TABLE I

Comparison of the averages of the execution times.

Methods	times [sec]
The proposed method using "pinv"	0.7580
The proposed method "mpinvl"	0.7011
The EVM using "pinv"	0.5963
The EVM using "mpinvl"	0.5829

VI. Conclusion

In this paper, by modifying the EVM, we have proposed an algorithm which can achieve the BD without calculating eigenvectors. It can be seen that our proposed algorithm provides better performance than the conventional EVM, but the average of execution time of the proposed algorithm is a little bit longer than the conventional EVM. Although there exists such a trade-off, we conclude that our proposed method is more useful for solving the BD problem, because we consider that the performance accuracy is most important for achieving the BD.

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