Robust Super-Exponential Methods for Blind Equalization of SISO Systems With Additive Gaussian Noise

Mitsuru Kawamoto*†§, Kiyotaka Kohno*¶, Yujiro Inouye*‡ and Asoke K. Nandi§
*Department of Electronic and Control Systems Engineering, Shimane University
1060 Nishikawatsu, Matsue, Shimane 690-8504 Japan
†Email: kawa@ecs.shimane-u.ac.jp, ¶Email: kohno@yonago-k.ac.jp, ‡Email: inouye@riko.shimane-u.ac.jp
§Department of Electrical Engineering and Electronics, The University of Liverpool,
Brownlow Hill, Liverpool, L69 3GJ, UK
Emails: {M.Kawamoto, A.Nandi}@liverpool.ac.uk

***Abstract*— The present paper deals with the blind equalization problem of a single-input single-output infinite impulse response (SISO-IIR) system with additive Gaussian noise. To solve the problem, we propose a new "super-exponential method" (SEM). The novel point of the proposed SEM is that even when Gaussian noise is added to the output of the system, the blind equalization can be achieved with as little influence of Gaussian noise as possible; hence the proposed SEM is referred to as a "robust super-exponential method" (RSEM). Simulation results show the validity of the proposed RSEM.**

I. Introduction

In applications such as (mobile or wireless) communications, an input signal often propagates through a multipath environment of unknown transfer function between the signal source and a receiver. Blind equalization is used to reconstruct the original input signal and/or to estimate the transfer function from the received signal.

Recently Shalvi and Weinstein proposed an attractive approach for the blind equalization of SISO system, which is called the super-exponential method (SEM) [6]. One of the attractive properties of the SEM is to converge to the desired solutions, which achieve the blind equalization, at super-exponential rate; hence the "super-exponential" method was named. However, the SEM has such a significant drawback that if the SEM is applied to the blind equalization in the presence of additive Gaussian noise, then the convergence of the SEM close to the desired solutions cannot be guaranteed [6]. In this paper, a new approach is proposed in order to overcome the drawback. In the proposed approach, only higher-order cumulants are used; consequently the proposed algorithm can be used to detect the desired solutions with as little the influence of Gaussian noise as possible, from which the proposed SEM is referred to as a *robust super-exponential method* (RSEM). Computer simulations are presented to demonstrate the validity of the proposed RSEM.

II. Problem Formulation and Assumptions

We consider a single-input single-output (SISO) system with an additive noise as described by

$$y(t) = \sum_{k=-\infty}^{\infty} h^{(k)} s(t-k) + n(t), \tag{1}$$

where $\{s(t)\}$ is an unobserved input sequence generated from a discrete-time stationary random process, $h^{(k)}$ is the impulse response of an unknown time-invariant system defined by $H(z) = \sum_{k=-\infty}^{\infty} h^{(k)} z^k$, $y(t)$ and $n(t)$ denote the output of the system and Gaussian noise, respectively. Fig. 1 illustrates a diagram of the basic problem. Namely, our objective in this paper is to propose a method for adjusting the equalizer $W(z) = \sum_{k=-L_1}^{L_2} w^{(k)} z^k$ so that $G(z) := W(z)H(z)$ becomes

$$\hat{G}(z) = \hat{W}(z)H(z) = cz^{k_1}, \tag{2}$$

even if the Gaussian noise $n(t)$ is included into the output $y(t)$, where c in cz^{k_1} is a nonzero complex number standing for a scale change and a phase shift, and the superscript "k_1" of z^{k_1} denotes an integer standing for a constant delay. We allow all of the above signals to be complex-valued.

To find the solution (2), we put the following assumptions on the system, the input signal, and the equalizer.

A1) The unknown system $H(z)$ is a stable, possibly nonminimum phase, linear time-invariant filter whose inverse (which may be noncausal and stable) $H(z)^{-1}$ exists.

A2) The input sequence $\{s(t)\}$ is a complex, zero-mean, non-Gaussian random process. Moreover, the process $\{s(t)\}$ is an i.i.d. process with a nonzero variance σ_s^2 and a nonzero $(p+$

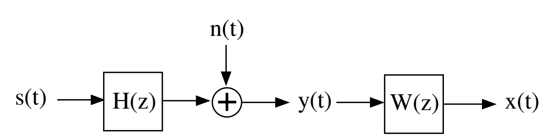

Fig. 1. The composite system of an unknown system and a filter.

$q+1$)st-order cumulants, κ_s defined as

$$\kappa_s = \mathrm{cum}\{\underbrace{s(t),\cdots,s(t)}_{p},\underbrace{s^*(t),\cdots,s^*(t)}_{q+1}\}, \tag{3}$$

where p and q are nonnegative integers such that $(p+q)\geq 2$, and $\mathrm{cum}\{s_1,\ s_2,\ \cdots,\ s_n\}$ denotes the nth-order (joint) cumulant of $s_1,\ s_2,\ \cdots,\ s_n$.

A3) The equalizer $W(z) = \sum_{l=L_1}^{L_2} w^{(k)} z^k$ is an FIR system of sufficient length $L = L_2 - L_1 + 1$ so that the truncation effect can be ignored.

The combined system response subject to the finite length restriction is

$$g^{(k)} = \sum_{l=L_1}^{L_2} w^{(l)} h^{(k-l)}. \tag{4}$$

In a vector notation, (4) can be rewritten as

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{w}, \tag{5}$$

where $\boldsymbol{g}$ is the possibly infinite vector of the combined system $\boldsymbol{g} = [\cdots, g^{(-1)}, g^{(0)}, g^{(1)}, \cdots]^T$, $\boldsymbol{w}$ is the L-column vector, that is, $\boldsymbol{w} = [w^{(L_1)}, w^{(L_1+1)}, \cdots, w^{(L_2)}]^T$, and $\boldsymbol{H} = [h_{kl}]$ is the matrix of L columns and possibly infinite number of rows, whose elements are $h_{kl} = h^{(k-l)}$, $k = -\infty, \cdots, \infty$, $l = L_1, (L_1+1), \cdots, L_2$.

III. Robust super-exponential methods (RSEMs)

A. Two-step iterative procedure of vector $\boldsymbol{g}$

To find the solution in (2), the following two-step iterative procedure with respect to the elements $g^{(k)}$'s of $\boldsymbol{g}$ is used:

$$g^{(k)[1]} = \frac{\kappa_s}{\gamma_s \alpha_k}(g^{(k)})^p (g^{(k)*})^q, \tag{6}$$

$$g^{(k)[2]} = g^{(k)[1]}/\sqrt{\sigma_x^2}, \tag{7}$$

where $(\cdot)^{[1]}$, $(\cdot)^{[2]}$ stand for the results of the first step and the second step per iteration, $g^{(k)}$ in the right-hand side of (6) is $g^{(k)[2]}$ at the previous step (note that at first iteration, $g^{(k)}$ in the right-hand side of (6) is the initial value of $g^{(k)}$), p and q are nonnegative integers such that $(p+q)\geq 2$, γ_s denotes the fourth-order cumulant of $s(t)$ defined by $\gamma_s := \mathrm{cum}\{s(t), s(t), s^*(t), s^*(t)\}$, α_k denotes a positive value (in subsection III.B, it will be shown how we choose the values of α_k's), the superscript $*$ denotes the complex conjugate, and σ_x^2 denotes the variance of $x(t)$, which is the output of the equalizer $W(z)$ (see Fig. 1).

The main difference between the two-step procedures in the conventional SEMs (e.g., [1], [2], [4], [5], [6], [8]) and the proposed one is the denominator of the first step, that is, the conventional first step procedures include the second-order cumulants of $s(t)$, whereas our proposed one, that is, (6) possesses only higher-order cumulants of $s(t)$.

Here, let $g^{(k)}(i)$ denote the value obtained in the ith cycle of the iterations of two steps (6) and (7). The important fact of the two-step procedure is that the infinite values $g^{(k)}(i)$ ($k = -\infty, \cdots, \infty$) converge to zero except for only one of the values as the iteration number i approaches infinity, that is, $i \to \infty$. The magnitude of the remaining one converges to a positive constant. This will be shown in the following theorem.

Theorem 1: Let $g^{(k)}(0)$ be an initial value for iterations of two steps (6) and (7) for each $k = -\infty, \cdots, \infty$. Let β_k be non-negative scalar defined as

$$\beta_k = |\frac{1}{\alpha_k}|^{\frac{1}{p+q-1}}. \tag{8}$$

Let k_0 be $k_0 = \arg\max_{k\in\{-\infty,\cdots,\infty\}} \beta_k |g^{(k)}(0)|$. Suppose the index k_0 is unique, that is, $\beta_{k_0}|g^{(k_0)}(0)| > \beta_k |g^{(k)}(0)|$ for any other $k \in \{-\infty, \cdots, \infty\}$, then as $i \to \infty$, it follows

$$\lim_{i\to\infty} |g^{(k)}(i)| = \begin{cases} 0 & \text{for } k \neq k_0, \\ \tilde{c} \neq 0 & \text{for } k = k_0, \end{cases} \tag{9}$$

where $\tilde{c}$ is a scalar positive constant.

Proof: From (6) and (8), choosing k_0 so that $g^{(k_0)}(i) \neq 0$, we obtain

$$\frac{|g^{(k)[1]}(i)|}{|g^{(k_0)[1]}(i)|} = \frac{\beta_k^{p+q-1}}{\beta_{k_0}^{p+q-1}} \frac{|g^{(k)[1]}(i-1)|^{p+q}}{|g^{(k_0)[1]}(i-1)|^{p+q}}, \tag{10}$$

where the integer i denotes the iteration time. Note that $|g^{(k)[1]}(i)|/|g^{(k_0)[1]}(i)|$ is not modified by the normalization of the second step. Therefore, it is possible to solve $|g^{(k)[2]}(i)|/|g^{(k_0)[2]}(i)|$ from the recursive formula (10), which yields

$$\frac{|g^{(k)[2]}(i)|}{|g^{(k_0)[2]}(i)|} = \frac{\beta_{k_0}}{\beta_k}\left(\frac{\beta_k}{\beta_{k_0}} \frac{|g^{(k)[2]}(0)|}{|g^{(k_0)[2]}(0)|}\right)^{(p+q)^i} \tag{11}$$

for any non-negative integer i. For $k_0 = \arg\max_k \beta_k |g^{(k)}(0)|$, one can see that all the other values $|g^{(k)[2]}(i)|$ ($k \neq k_0$) quickly become small compared to $|g^{(k_0)[2]}(i)|$. Taking into account the normalization of the second step, this means that $|g^{(k_0)[2]}(i)| \neq 0$ and $|g^{(k)[2]}(i)| \to 0$ for all $k \neq k_0$. This implies that the infinite iteration of two steps (6) and (7) gives (9). Moreover, the equation (11) along with the normalization of the second step means that the sequence $\{g^{(k)}(i)\}$ converges to a desired value at a super-exponential rate for all $k = -\infty, \cdots, \infty$. ∎

B. Two-step iterative procedure for $\boldsymbol{w}$

To find the solution $\hat{W}(z)$ in (2), we adjust the elements of the vector $\boldsymbol{w}$ so that $\boldsymbol{g} = \boldsymbol{H}\boldsymbol{w}$ is equal to the vector $\boldsymbol{\delta}^{(k_1)}$ whose nth element is $c\delta(n-k_1)$ for some fixed k_1, where $\delta(t)$ is the Kronecker delta function and k_1 is an integer standing for the same time shift as k_1 in (2). However, since $\boldsymbol{w}$ is of finite length, it may be only required that $\boldsymbol{w}$ is chosen to minimize the distance (norm) between $\boldsymbol{H}\boldsymbol{w}$ and $\boldsymbol{\delta}^{(k_1)}$. Hence, in order to derive an algorithm with respect to $\boldsymbol{w}$, we consider the following weighted least squares problem:

$$\min_{\boldsymbol{w}} (\boldsymbol{H}\boldsymbol{w} - \boldsymbol{g})^{T*} \boldsymbol{\Lambda} (\boldsymbol{H}\boldsymbol{w} - \boldsymbol{g}). \tag{12}$$

Here, $\boldsymbol{\Lambda}$ is a diagonal matrix whose diagonal elements all are positive values. The solution is known to be given by

$$\boldsymbol{w} = (\boldsymbol{H}^{T*}\boldsymbol{\Lambda}\boldsymbol{H})^{-1}\boldsymbol{H}^{T*}\boldsymbol{\Lambda}\boldsymbol{g}. \tag{13}$$

Note that from assumption A1), $\boldsymbol{H}^{T*}\boldsymbol{\Lambda}\boldsymbol{H}$ is invertible for any L, because $\boldsymbol{H}$ is of full column rank and $\boldsymbol{\Lambda}$ is a nonsingular diagonal matrix (this fact is also mentioned in [6] (p. 508, line 10) without proof). The update rules of $\boldsymbol{w}$ in the conventional and the proposed SEMs are based on (13).

In the conventional SEMs ([1], [2], [4], [5], [6], [8]), the positive diagonal elements of $\boldsymbol{\Lambda}$ in (13) are set to 1 or the variance of the input $s(t)$. This means that $\boldsymbol{H}^{T*}\boldsymbol{\Lambda}\boldsymbol{H}$ is calculated by the second-order statistics of the output $y(t)$. We consider that this is the reason why the conventional SEMs are sensitive to Gaussian noise.

In what follows, we shall show that $\boldsymbol{H}^{T*}\boldsymbol{\Lambda}\boldsymbol{H}$ in (13) can be applied to a set of fourth-order cumulants of the output $y(t)$, if we choose appropriately a diagonal matrix $\boldsymbol{\Lambda}$ in (12). To this end, as the diagonal elements λ_k $(k = -\infty, \cdots, \infty)$ of $\boldsymbol{\Lambda}$, we choose the λ_k's expressed as

$$\lambda_k := \mathrm{sign}(\gamma_s)\gamma_s\tilde{\alpha}, \qquad k = -\infty, \cdots, \infty, \tag{14}$$

$$\tilde{\alpha} := \textstyle\sum_{l=-\infty}^{\infty} |h^{(l)}|^2, \tag{15}$$

where sign(γ) in (14) denotes the sign of γ, that is, sign(γ) = 1 if $\gamma > 0$, sign(γ) = 0 if $\gamma = 0$, and sign(γ) = -1 if $\gamma < 0$, and $h^{(l)}$ in (15) denotes the impulse response of $H(z)$. [We note that the elements of $\boldsymbol{\Lambda}$ are positive values.] From (14) and (15), $\boldsymbol{\Lambda}$ can be expressed as $\dot{\boldsymbol{I}}\tilde{\boldsymbol{\Lambda}}$, where $\dot{\boldsymbol{I}}$ is a diagonal matrix whose all elements are sign(γ_s), that is, +1 or -1, and $\tilde{\boldsymbol{\Lambda}}$ is also a diagonal matrix whose elements are $\gamma_s\tilde{\alpha}$. Then substituting $\dot{\boldsymbol{I}}\tilde{\boldsymbol{\Lambda}}$ into $\boldsymbol{\Lambda}$ in (13), the right-hand side of (13) becomes

$$(\boldsymbol{H}^{T*}\dot{\boldsymbol{I}}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H})^{-1}\boldsymbol{H}^{T*}\dot{\boldsymbol{I}}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}. \tag{16}$$

From (16), we obtain

$$(\boldsymbol{H}^{T*}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H})^{-1}\boldsymbol{H}^{T*}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}, \tag{17}$$

because $\dot{\boldsymbol{I}}$ is a diagonal matrix whose all elements are either $+1$ or -1.

Here, $\boldsymbol{H}^{T*}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H}$ in (17) can be expressed by the fourth-order cumulants matrix of $y(t)$, which is defined by $[\boldsymbol{C}_{y,l}^{(4)}]_{r_1,r_2}$ = cum$\{y(t-r_1), y^*(t-r_2), y(t-l), y^*(t-l)\}$ [7], that is,

$$\boldsymbol{H}^{T*}\tilde{\boldsymbol{\Lambda}}\boldsymbol{H} := \textstyle\sum_{l=-\infty}^{\infty}\boldsymbol{C}_{y,l}^{(4)}, \tag{18}$$

where $[X]_{r_1,r_2}$ denotes the (r_1, r_2)th element of the $L \times L$ matrix X, in which r_i's take the values of $L_1, (L_1{+}1), \cdots, L_2$. As for $\boldsymbol{H}^{T*}\tilde{\boldsymbol{\Lambda}}\boldsymbol{g}$ in (17), by using (6) with $\alpha_k = \tilde{\alpha}$ in (15) and the similar way as in [2], it can be given by

$$\boldsymbol{d} := [d_{L_1}, d_{L_1+1}, \cdots, d_{L_2}]^T, \tag{19}$$

where d_l's are given by d_l = cum$\{\underbrace{x(t), \cdots, x(t)}_{p}, \underbrace{x^*(t), \cdots, x^*(t)}_{q} y^*(t-j)\}$ $(l = L_1, L_1+1, \cdots, L_2)$. Therefore, it can be seen from (18) and (19) that the right-hand side of (13) can be calculated by the fourth-order statistics of the output $y(t)$, provided that $\boldsymbol{\Lambda}$ in (12) is replaced by $\dot{\boldsymbol{I}}\tilde{\boldsymbol{\Lambda}}$. Then, (17) can be expressed as

$$\boldsymbol{w}^{[1]} = \boldsymbol{R}^{-1}\boldsymbol{d}, \tag{20}$$

where $\boldsymbol{R}$:= $\sum_{l=-\infty}^{\infty}\boldsymbol{C}_{y,l}^{(4)}$. It can be easily shown that the second step (7) is expressed as

$$\boldsymbol{w}^{[2]} := \boldsymbol{w}^{[1]}/\sqrt{\sigma_x^2}. \tag{21}$$

Therefore, (20) and (21) are our proposed two steps to modify $\boldsymbol{w}$.

From (20), it can be seen that since the update procedure of $\boldsymbol{w}$ consists of only higher-order cumulants of $y(t)$, then the two-step procedure (20) and (21) becomes less sensitive to Gaussian noise. [Note that since (21) is only used to normalize $\boldsymbol{w}$, even if σ_x^2 is a second-order statistic, there is less effect of Gaussian noise for finding the desired solution $\hat{\boldsymbol{w}}$, that is, $\boldsymbol{H}\hat{\boldsymbol{w}}$=$\boldsymbol{\delta}^{(k_1)}$.] This is a novel key point of our proposed SEM, from which the proposed method is referred to as a *robust super-exponential method* (RSEM).

IV. COMPUTER SIMULATIONS

To demonstrate the validity of the proposed method, many computer simulations were conducted. Some results are shown in this section. The unknown system $H(z)$ was set to be a filter of length 7 with the impulse responses (0.4, 1, -0.7, 0.6, 0.3, -0.4, 0.1), which is the same system as in [6]. We used an equalizer of length L = 16 which was initialized to $\boldsymbol{w}(0)$ = $[0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0]^T$, which is also the same situation as in [6]. The input $s(t)$ of the system $H(z)$ was sub-Gaussian which takes one of two values, -1 and 1 with equal probability 1/2. The parameters p and q in (6) were set to be p=2 and q=1, respectively, that is, κ_s in (3) was the fourth-order cumulants of $s(t)$. Then the value of κ_s is -2. The Gaussian noise $n(t)$ with its variance σ_n^2 was included in the output $y(t)$ at various SNR levels. The SNR is, for convenience, defined as SNR := 10 $\log_{10}(\sigma_s^2/\sigma_n^2)$, where σ_s^2 is the variance of $s(t)$ and is equal to 1. As a measure of performance, we used the intersymbol interference (ISI) defined in the logarithmic (dB) scale by

$$\mathrm{ISI} = 10\log_{10}\frac{\sum_k |g^{(k)}|^2 - |g_{\max}|^2}{|g_{\max}|^2} \tag{22}$$

where $g_{\max}$ is the component of $g^{(k)}$ having the maximal absolute value (the leading tap). The value of ISI becomes $-\infty$ if $g^{(k)}$'s satisfying (2) are obtained and hence a large negative value of ISI indicates the proximity to the desired solution.

For comparison, the SEM proposed in [6] was used. Fig. 2 shows the performances of the proposed RSEM and the SEM proposed by Shalvi and Weinstein, in the cases where the SNR levels were taken to be 0 dB (σ_n^2=1) and ∞ dB (σ_n^2=0), and for each iteration, the vector $\boldsymbol{w}$ for each method was modified every each data set of the following three cases (i) 5,000 samples (Fig. 2(i)), (ii) 10,000 samples (Fig. 2(ii)), and (iii) 30,000 samples (Fig. 2(iii)), by using the vector $\boldsymbol{d}$ corresponding to each method, where the details of the calculations of the vector $\boldsymbol{d}$ and the matrix $\boldsymbol{R}$ in (20) will be found in [3]. The vertical and horizontal axes in Fig. 2 represent the average ISI denoted by <ISI> and the number of iterations, respectively. For each SNR and in each of the

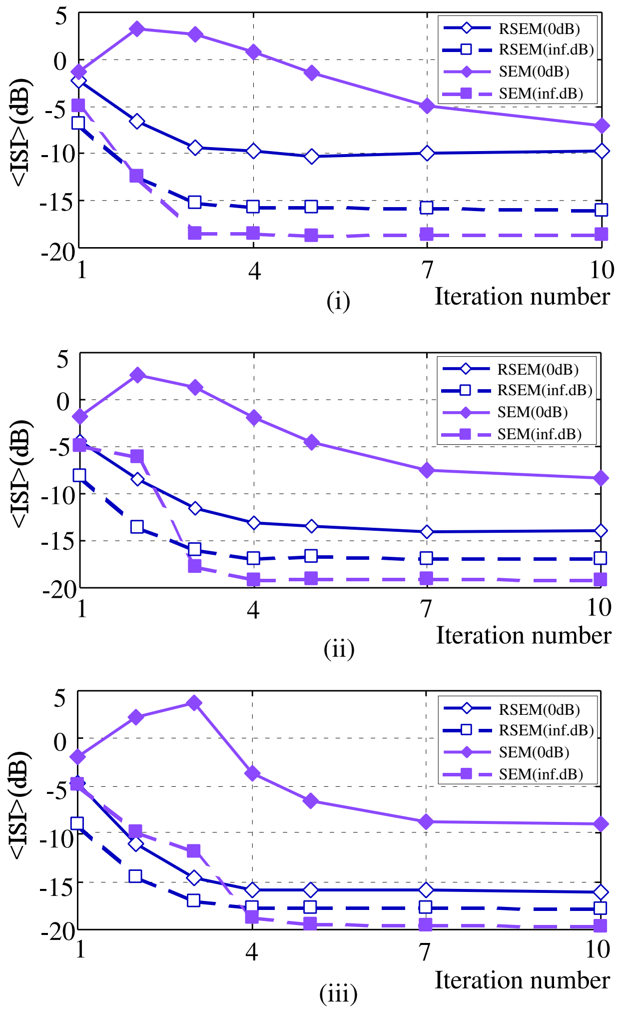

Fig. 2. The average performances of the RSEM and the SEM with varying number of iterations, in the three cases of (i) 5000 samples, (ii) 10000 samples, (iii) 30000 samples, in each of which SNR = 0 dB and ∞ dB.

three cases, the methods were tested using 100 Monte Carlo trials, and the <ISI> for each iteration was computed with the results in the trials. Fig. 3 shows the average performances of the RSEM and the SEM at the 10th iteration for Fig. 2.

From Fig. 2 and Fig. 3, it can be seen that for each data set, in the case of SNR = 0 dB, as the number of data samples, which are needed to estimate the cumulants, increases, the RSEM gives better performance, whereas the performance of the SEM hardly changes. In the case of SNR = ∞ dB, however, it can be seen from Fig. 2 and Fig. 3 that the performance of the RSEM becomes worse than that of the SEM. This has resulted from the fact that it is difficult to estimate the fourth-order cumulants of $\boldsymbol{R}$ in (20) with high accuracy, compared with the second-order cumulants of $\boldsymbol{R}$ in the SEM. Namely, this means that in order to estimate the matrix $\boldsymbol{R}$ in (20) with high accuracy, the data more than 30000 samples are needed. We consider that the property that a lot of data samples are needed to estimate the fourth-order cumulants in the RSEM is a drawback of the RSEM. Incidentally, we conform that when the matrix $\boldsymbol{R}$ possesses its theoretical value, the performance of the RSEM becomes similar to that of the SEM.

From these results, we can see that although the performance of the RSEM depends on the accuracy of the estimate of the higher-order cumulants, especially the matrix $\boldsymbol{R}$ in (20), in the case of SNR = 0 dB (σ_n^2 = 1), the RSEM provides significantly better performance than the SEM. Therefore, we consider that the RSEM is effective for the blind equalization of the system with additive Gaussian noise as shown in Fig. 1.

V. Conclusions

We have proposed an SEM for solving a blind equalization problem, which is referred to as a *robust super-exponential method* (RSEM). The RSEM is robust against Gaussian noise, which means that the RSEM can be used to estimate the inverse of the unknown transfer function $H(z)$, even if Gaussian noise is added to the output of $H(z)$ (see (1)). This is a novel property of the proposed method, not possessed by the conventional SEMs. Computer simulations have demonstrated the validity of the RSEM.

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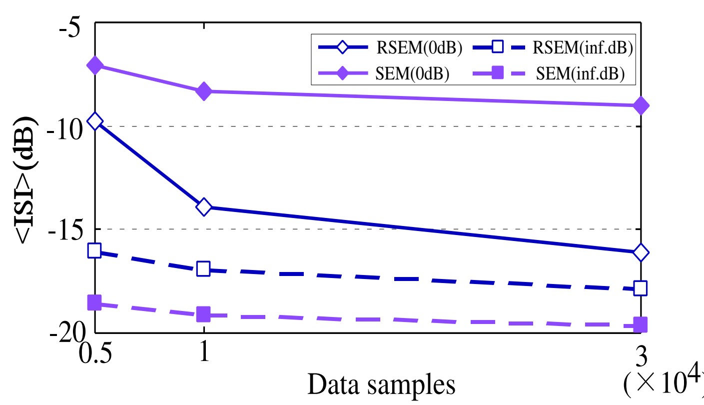

Fig. 3. The average performances of the RSEM and the SEM with varying number of iterations at the 10th iteration for Fig. 2.