An Eigenvector Algorithm with Reference Signals Using a Deflation Approach for Blind Deconvolution

Mitsuru Kawamoto[1], Yujiro Inouye[2], Kiyotaka Kohno[2], and Takehide Maeda[2]

[1] National Institute of Advanced Industrial Science and Technology (AIST), Central 2, 1-1-1 Umezono, Tsukuba, Ibaraki, 305-8568, Japan
m.kawamoto@aist.go.jp
http://staff.aist.go.jp/m.kawamoto/

[2] Department of Electronic and Control Systems Engineering, Shimane University, 1060 Nishikawatsu, Matsue, 690-8504, Japan
inouye@riko.shimane-u.ac.jp,
kohno@yonago-k.ac.jp, maeda@helen.ocn.ne.jp

Abstract. We propose an eigenvector algorithm (EVA) with reference signals for blind deconvolution (BD) of multiple-input multiple-output infinite impulse response (MIMO-IIR) channels. Differently from the conventional EVAs, each output of a deconvolver is used as a reference signal, and moreover the BD can be achieved without using whitening techniques. The validity of the proposed EVA is shown comparing with our conventional EVA.

1 Introduction

This paper deals with a blind deconvolution (BD) problem for a multiple-input and multiple-output (MIMO) infinite impulse response (IIR) channels. To solve this problem, we use eigenvector algorithms (EVAs) [6,7,12]. The first proposal of the EVA was done by Jelonnek et al. [6]. They have proposed the EVA for solving blind equalization (BE) problems of single-input single-output (SISO) channels or single-input multiple-output (SIMO) channels. In [12], several procedures for the blind source separation (BSS) of instantaneous mixtures, using the generalized eigenvalue decomposition (GEVD), have been introduced. Recently, the authors have proposed an EVA which can solve BSS problems in the case of MIMO static systems (instantaneous mixtures) [8]. Moreover, based on the idea in [8], an EVA was derived for MIMO-IIR channels (convolutive mixtures) [9].

In the EVAs in [8,9], an idea of using reference signals was adopted. Researches applying this idea to solving blind signal processing (BSP) problems, such as the BD, the BE, the BSS, and so on, have been made by Jelonnek et al. (e.g., [6]), Adib et al. (e.g., [2]), Rhioui et al. [13], and Castella, et al. [3]. In [8,9], differently from the conventional methods, only one reference signal was utilized for recovering all the source signals simultaneously.

However, the EVA in [9] has difference performances for a different choice of the reference signal (see section 4), and in order to recover all source signals, it

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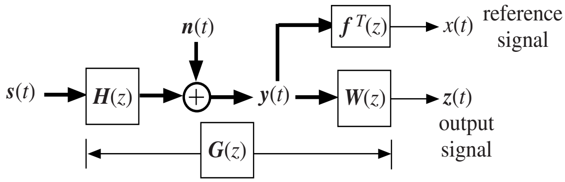

Fig. 1. The composite system of an unknown system and a deconvolver, and a reference system

must be taken into account how to select appropriate eigenvectors from the set of eigenvectors calculated by the EVA. In this paper, in order to circumvent such a tedious (or nasty) task, the output of a deconvolver which is used to recover source signals is used as a reference signal. Accordingly, deflation techniques are needed to recover all source signals. The method proposed in [3] is almost same as the proposed EVA. However, the proposed EVA can achieve the BD without using whitening techniques. Moreover, the proposed EVA provides good performances compared with our conventional EVA [9] (see section 4).

The present paper uses the following notation: Let Z denote the set of all integers. Let C denote the set of all complex numbers. Let $\boldsymbol{C}^n$ denote the set of all n-column vectors with complex components. Let $\boldsymbol{C}^{m\times n}$ denote the set of all $m \times n$ matrices with complex components. The superscripts T, $*$, and H denote, respectively, the transpose, the complex conjugate, and the complex conjugate transpose (Hermitian) of a matrix. The symbols block-diag$\{\cdots\}$ and diag$\{\cdots\}$ denote respectively a block diagonal and a diagonal matrices with the block diagonal and the diagonal elements $\{\cdots\}$. The symbol cum$\{x_1,x_2,x_3,x_4\}$ denotes a fourth-order cumulant of x_i's. Let $i = \overline{1, n}$ stand for $i = 1, 2, \cdots, n$.

2 Problem Formulation and Assumptions

We consider a MIMO channel with n inputs and m outputs as described by

$$\boldsymbol{y}(t) = \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k) + \boldsymbol{n}(t), \quad t \in Z, \tag{1}$$

where $\boldsymbol{s}(t)$ is an n-column vector of input (or source) signals, $\boldsymbol{y}(t)$ is an m-column vector of channel outputs, $\boldsymbol{n}(t)$ is an m-column vector of Gaussian noises, and $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ impulse response matrix sequence. The transfer function of the channel is defined by $\boldsymbol{H}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} z^k$, $z \in C$.

To recover the source signals, we process the output signals by an $n \times m$ deconvolver (or equalizer) $\boldsymbol{W}(z)$ described by

$$\begin{aligned} \boldsymbol{z}(t) &= \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{y}(t-k) \\ &= \textstyle\sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} \boldsymbol{s}(t-k) + \sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)} \boldsymbol{n}(t-k), \end{aligned} \tag{2}$$

where $\{\boldsymbol{G}^{(k)}\}$ is the impulse response matrix sequence of $\boldsymbol{G}(z) := \boldsymbol{W}(z)\boldsymbol{H}(z)$, which is defined by $\boldsymbol{G}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{G}^{(k)} z^k$, $z \in C$. The cascade connection of the unknown system and the deconvolver is illustrated in Fig. 1.

Here, we put the following assumptions on the channel, the source signals, the deconvolver, and the noises.

A1) The transfer function $\boldsymbol{H}(z)$ is stable and has full column rank on the unit circle $|z| = 1$, where the assumption **A1)** implies that the unknown system has less inputs than outputs, i.e., $n \leq m$, and there exists a left stable inverse of the unknown system.
A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a complex, zero-mean and non-Gaussian random vector process with element processes $\{s_i(t)\}$, $i = \overline{1, n}$ being mutually independent. Each element process $\{s_i(t)\}$ is an i.i.d. process with a variance $\sigma_{s_i}^2 \neq 0$ and a nonzero fourth-order cumulant $\gamma_i \neq 0$ defined as

$$\gamma_i = \mathrm{cum}\{s_i(t), s_i(t), s_i^*(t), s_i^*(t)\} \neq 0. \tag{3}$$

A3) The deconvolver $\boldsymbol{W}(z)$ is an FIR channel of sufficient length L so that the truncation effect can be ignored.
A4) The noise sequence $\{\boldsymbol{n}(t)\}$ is a zero-mean, Gaussian vector stationary process whose component processes $\{n_j(t)\}$, $j = \overline{1, m}$ have nonzero variances $\sigma_{n_j}^2$, $j = \overline{1, m}$.
A5) The two vector sequences $\{\boldsymbol{n}(t)\}$ and $\{\boldsymbol{s}(t)\}$ are mutually statistically independent.

Under **A3)**, the impulse response $\{\boldsymbol{G}^{(k)}\}$ of the cascade system is given by

$$\boldsymbol{G}^{(k)} := \textstyle\sum_{\tau=L_1}^{L_2} \boldsymbol{W}^{(\tau)} \boldsymbol{H}^{(k-\tau)}, \quad k \in Z, \tag{4}$$

where the length $L := L_2 - L_1 + 1$ is taken to be sufficiently large. In a vector form, (4) can be written as

$$\tilde{\boldsymbol{g}}_i = \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i, \quad i = \overline{1, n}, \tag{5}$$

where $\tilde{\boldsymbol{g}}_i$ is the column vector consisting of the ith output impulse response of the cascade system defined by $\tilde{\boldsymbol{g}}_i := [\boldsymbol{g}_{i1}^T, \boldsymbol{g}_{i2}^T, \cdots, \boldsymbol{g}_{in}^T]^T$,

$$\boldsymbol{g}_{ij} := [\cdots, g_{ij}(-1), g_{ij}(0), g_{ij}(1), \cdots]^T, \quad j = \overline{1, n} \tag{6}$$

where $g_{ij}(k)$ is the (i, j)th element of matrix $\boldsymbol{G}^{(k)}$, and $\tilde{\boldsymbol{w}}_i$ is the mL-column vector consisting of the tap coefficients (corresponding to the ith output) of the deconvolver defined by $\tilde{\boldsymbol{w}}_i := \left[\boldsymbol{w}_{i1}^T, \boldsymbol{w}_{i2}^T, \cdots, \boldsymbol{w}_{im}^T\right]^T \in \boldsymbol{C}^{mL}$,

$$\boldsymbol{w}_{ij} := [w_{ij}(L_1), w_{ij}(L_1 + 1), \cdots, w_{ij}(L_2)]^T \in \boldsymbol{C}^L, \tag{7}$$

$j = \overline{1, m}$, where $w_{ij}(k)$ is the (i, j)th element of matrix $\boldsymbol{W}^{(k)}$, and $\tilde{\boldsymbol{H}}$ is the $n \times m$ block matrix whose (i, j)th block element $\boldsymbol{H}_{ij}$ is the matrix (of L columns and possibly infinite number of rows) with the (l, r)th element $[\boldsymbol{H}_{ij}]_{lr}$ defined by $[\boldsymbol{H}_{ij}]_{lr} := h_{ji}(l - r)$, $l = 0, \pm 1, \pm 2, \cdots$, $r = \overline{L_1, L_2}$, where $h_{ij}(k)$ is the (i, j)th element of the matrix $\boldsymbol{H}^{(k)}$.

In the multichannel blind deconvolution problem, we want to adjust $\tilde{\boldsymbol{w}}_i$'s (i $= \overline{1, n}$) so that

$$[\tilde{\boldsymbol{g}}_1, \cdots, \tilde{\boldsymbol{g}}_n] = \tilde{\boldsymbol{H}}[\tilde{\boldsymbol{w}}_1, \cdots, \tilde{\boldsymbol{w}}_n] = [\tilde{\boldsymbol{\delta}}_1, \cdots, \tilde{\boldsymbol{\delta}}_n]\boldsymbol{P}, \tag{8}$$

where $\boldsymbol{P}$ is an $n \times n$ permutation matrix, and $\tilde{\boldsymbol{\delta}}_i$ is the n-block column vector defined by

$$\tilde{\boldsymbol{\delta}}_i := [\boldsymbol{\delta}_{i1}^T, \boldsymbol{\delta}_{i2}^T, \ldots, \boldsymbol{\delta}_{in}^T]^T, \qquad i = \overline{1, n} \tag{9}$$

$$\boldsymbol{\delta}_{ij} := \begin{cases} \hat{\boldsymbol{\delta}}_i, & \text{if} \quad i = j, \\ (\cdots, 0, 0, 0, \cdots)^T, & \text{otherwise.} \end{cases} \tag{10}$$

Here, $\hat{\boldsymbol{\delta}}_i$ is the column vector (of infinite elements) whose rth element $\hat{\delta}_i(r)$ is given by $\hat{\delta}_i(r) = d_i \delta(r - k_i)$, where $\delta(t)$ is the Kronecker delta function, d_i is a complex number standing for a scale change and a phase shift, and k_i is an integer standing for a time shift.

3 Eigenvector Algorithms (EVAs)

3.1 Analysis of Eigenvector Algorithms with Reference Signals for MIMO-IIR Channels

In order to solve the BD problem, the following cross-cumulant between $z_i(t)$ and a reference signal $x(t)$ (see Fig. 1) is defined;

$$D_{z_i x} = \text{cum}\{z_i(t), z_i^*(t), x(t), x^*(t)\}, \tag{11}$$

where $z_i(t)$ is the ith element of $\boldsymbol{z}(t)$ in (2) and the reference signal $x(t)$ is given by $\boldsymbol{f}^T(z)\boldsymbol{y}(t)$, using an appropriate filter $\boldsymbol{f}(z)$. The filter $\boldsymbol{f}(z)$ is called a *reference system*. Let $\boldsymbol{a}(z) := \boldsymbol{H}^T(z)\boldsymbol{f}(z) = [a_1(z), a_2(z), \cdots, a_n(z)]^T$, then $x(t) = \boldsymbol{f}^T(z)\boldsymbol{H}(z)\boldsymbol{s}(t) = \boldsymbol{a}^T(z)\boldsymbol{s}(t)$. The element $a_i(z)$ of the filter $\boldsymbol{a}(z)$ is defined as $a_i(z) = \sum_{k=-\infty}^{\infty} a_i(k) z^k$ and the reference system $\boldsymbol{f}(z)$ is an m-column vector whose elements are $f_j(z) = \sum_{k=L_1}^{L_2} f_j(k) z^k$, $j = \overline{1, m}$.

Jelonnek et al. [6] have shown in the single-input case that by the Lagrangian method, the maximization of $|D_{z_i x}|$ under $\sigma_{z_i}^2 = \sigma_{s_{\rho_i}}^2$ leads to a closed-form solution expressed as a generalized eigenvector problem, where $\sigma_{z_i}^2$ and $\sigma_{s_{\rho_i}}^2$ denote the variances of the output $z_i(t)$ and a source signal $s_{\rho_i}(t)$, respectively, and ρ_i is one of integers $\{1, 2, \cdots, n\}$ such that the set $\{\rho_1, \rho_2, \cdots, \rho_n\}$ is a permutation of the set $\{1, 2, \cdots, n\}$. In our case, $D_{z_i x}$ and $\sigma_{z_i}^2$ can be expressed in terms of the vector $\tilde{\boldsymbol{w}}_i$ as, respectively,

$$D_{z_i x} = \tilde{\boldsymbol{w}}_i^H \tilde{\boldsymbol{B}} \tilde{\boldsymbol{w}}_i, \quad \sigma_{z_i}^2 = \tilde{\boldsymbol{w}}_i^H \tilde{\boldsymbol{R}} \tilde{\boldsymbol{w}}_i, \tag{12}$$

where $\tilde{\boldsymbol{B}}$ is the $m \times m$ block matrix whose (i, j)th block element $\boldsymbol{B}_{ij}$ is the matrix with the (l, r)th element $[\boldsymbol{B}_{ij}]_{lr}$ calculated by cum$\{y_i^*(t - L_1 - l + 1), y_j(t - L_1 - r + 1), x^*(t), x(t)\}$ $(l, r = \overline{1, L})$ and $\tilde{\boldsymbol{R}} = E[\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)]$ is the covariance matrix of m-block column vector $\tilde{\boldsymbol{y}}(t)$ defined by

$$\tilde{\boldsymbol{y}}(t) := \left[\boldsymbol{y}_1^T(t), \boldsymbol{y}_2^T(t), \cdots, \boldsymbol{y}_m^T(t)\right]^T \in \boldsymbol{C}^{mL}, \tag{13}$$

$$\boldsymbol{y}_j(t) := [y_j(t\text{-}L_1), y_j(t\text{-}L_1\text{-}1), \cdots, y_j(t\text{-}L_2)]^T \in \boldsymbol{C}^L, \tag{14}$$

$j = \overline{1, m}$. Therefore, by the similar way to as in [6], the maximization of $|D_{z_i x}|$ under $\sigma^2_{z_i} = \sigma^2_{s_{\rho_i}}$ leads to the following generalized eigenvector problem;

$$\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i. \tag{15}$$

Moreover, Jelonnek et al. have shown that the eigenvector corresponding to the maximum magnitude eigenvalue of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}$ becomes the solution of the blind equalization problem in [6], which is referred to as an *eigenvector algorithm* (EVA). Note that since Jelonnek et al. have dealt with SISO-IIR channels or SIMO-IIR channels, the constructions of $\tilde{\boldsymbol{B}}$, $\tilde{\boldsymbol{w}}_i$, and $\tilde{\boldsymbol{R}}$ in (15) are different from those proposed in [6,7]. In this paper, we want to show how the eigenvector algorithm (15) works for the BD of the MIMO-IIR channel (1).

To this end, we use the following equalities;

$$\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{H}}, \quad \tilde{\boldsymbol{B}} = \tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{\Lambda}} \tilde{\boldsymbol{H}}, \tag{16}$$

where $\tilde{\boldsymbol{\Sigma}}$ is the block diagonal matrix defined by

$$\tilde{\boldsymbol{\Sigma}} := \text{block-diag}\{\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, \cdots, \boldsymbol{\Sigma}_n\}, \tag{17}$$

$$\boldsymbol{\Sigma}_i := \text{diag}\{\cdots, \sigma^2_{s_i}, \sigma^2_{s_i}, \sigma^2_{s_i}, \cdots\}, \quad i = \overline{1, n}, \tag{18}$$

and $\tilde{\boldsymbol{\Lambda}}$ is the block diagonal matrix defined by

$$\tilde{\boldsymbol{\Lambda}} := \text{block-diag}\{\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \cdots, \boldsymbol{\Lambda}_n\}, \tag{19}$$

$$\boldsymbol{\Lambda}_i := \text{diag}\{\cdots, |a_i(-1)|^2\gamma_r, |a_i(0)|^2\gamma_i, |a_i(1)|^2\gamma_i, \cdots\}, \tag{20}$$

$i = \overline{1, n}$. Since both $\tilde{\boldsymbol{\Sigma}}$ and $\tilde{\boldsymbol{\Lambda}}$ become diagonal, (16) shows that the two matrices $\tilde{\boldsymbol{R}}$ and $\tilde{\boldsymbol{B}}$ are simultaneously diagonalizable.

Here, let the eigenvalues of the diagonal matrix $\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\boldsymbol{\Lambda}}$ is denoted by

$$\lambda_i(k) := |a_i(k)|^2\gamma_i/\sigma^2_{s_i}, \qquad i = \overline{1, n}, \quad k \in Z. \tag{21}$$

We put the following assumption on the eigenvalues $\lambda_i(k)'s$.
A6) All the eigenvalues $\lambda_i(k)'s$ are distinct for $i = \overline{1, n}$ and $k \in Z$.

Theorem 1. *Suppose the noise term $\boldsymbol{n}(t)$ is absent and the length L of the deconvolver is infinite (that is, $L_1 = -\infty$ and $L_2 = \infty$). Then, under the assumptions* **A1)** *through* **A6)***, the n eigenvector $\tilde{\boldsymbol{w}}_i$'s corresponding to the n nonzero eigenvalues $\lambda_i(k)'s$ of matrix $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}$ for $i = \overline{1, n}$ and an arbitrary $k \in Z$ become the vectors $\tilde{\boldsymbol{w}}_i$'s satisfying (8).*

Outline of the proof: Based on (15), we consider the following eigenvector problem;

$$\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{w}}_i. \tag{22}$$

Then, from (16), (22) becomes

$$(\tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{H}})^{\dagger} \tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{\Lambda}} \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{w}}_i. \tag{23}$$

Under $L_1 = -\infty$ and $L_2 = \infty$, we have the following equations;

$$(\tilde{\boldsymbol{H}}^H \tilde{\boldsymbol{\Sigma}} \tilde{\boldsymbol{H}})^\dagger = \tilde{\boldsymbol{H}}^\dagger \tilde{\boldsymbol{\Sigma}}^\dagger \tilde{\boldsymbol{H}}^{H\dagger}, \ \ \tilde{\boldsymbol{H}}^{H\dagger} \tilde{\boldsymbol{H}}^H = \boldsymbol{I}, \tag{24}$$

which are shown in [11] along with their proofs. Then it follows from (23) and (24);

$$\tilde{\boldsymbol{H}}^\dagger \tilde{\boldsymbol{\Sigma}}^{-1} \tilde{\boldsymbol{\Lambda}} \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{w}}_i. \tag{25}$$

Multiplying (25) by $\tilde{\boldsymbol{H}}$ from the left side and using (24), (25) becomes

$$\tilde{\boldsymbol{\Sigma}}^{-1} \tilde{\boldsymbol{\Lambda}} \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i = \lambda_i \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i. \tag{26}$$

By (22), $\tilde{\boldsymbol{\Sigma}}^{-1} \tilde{\boldsymbol{\Lambda}}$ is a diagonal matrix with diagonal elements $\lambda_i(k)$, $i = \overline{1,n}$ and $k \in Z$, and thus (22) and (26) show that its diagonal elements $\lambda_i(k)'s$ are eigenvalues of matrix $\tilde{\boldsymbol{R}}^\dagger \tilde{\boldsymbol{B}}$. Here we use the following fact;

$$\lim_{L\to\infty} (\text{rank } \tilde{\boldsymbol{R}})/L = n, \tag{27}$$

which is shown in [10] and its proof is found in [4]. Using this fact, the other remaining eigenvalues of $\tilde{\boldsymbol{R}}^\dagger \tilde{\boldsymbol{B}}$ are all zero. From the assumption **A6)**, the n nonzero eigenvalues $\lambda_i(k) \neq 0$, $i = \overline{1,n}$, obtained by (26), that is, the n nonzero eigenvectors $\tilde{\boldsymbol{w}}_i$, $i = \overline{1,n}$, corresponding to n nonzero eigenvalues $\lambda_i(k) \neq 0$, $i = \overline{1,n}$, obtained by (22) become n solutions of the vectors $\tilde{\boldsymbol{w}}_i$ satisfying (8).

3.2 How to Choose a Reference Signal

In [9], a reference system $\boldsymbol{f}(z)$ is appropriately chosen, and then all source signals can be recovered simultaneously from the observed signals. However, the performances obtained by the EVA in [9] change with the way of choosing a reference system (see section 4) and moreover, the EVA has such a complicated task that the way of selecting appropriate eigenvectors from the set of eigenvectors calculated from the EVA must be taken into account.

In this paper, by adopting $x_i(t) = \tilde{\boldsymbol{w}}_i^T \tilde{\boldsymbol{y}}_i(t)$ as a reference signal, we want to circumvent such a tedious (or nasty) task. To this end, (11) can be reformulated as

$$D_{z_i x_i} = \text{cum}\{z_i(t), z_i^*(t), x_i(t), x_i^*(t)\}, \ i = \overline{1,n}, \tag{28}$$

The vector $\tilde{\boldsymbol{w}}_i$ in $x_i(t)$ is given by an eigenvector obtained from the EVA at the previous time, that is, $x_i(t) = \tilde{\boldsymbol{w}}_i^T(t-1)\tilde{\boldsymbol{y}}_i(t)$, where the value of $\tilde{\boldsymbol{w}}_i^T(t-1)$ is assumed to be fixed. By using $x_i(t)$, the matrix $\tilde{\boldsymbol{B}}$ is calculated, which is denoted by $\tilde{\boldsymbol{B}}_i(t)$, and then the eigenvector $\tilde{\boldsymbol{w}}_i^T(t)$ at time t can be obtained from the EVA using $\tilde{\boldsymbol{B}}_i(t)$. By repeating this operation, the BD can be achieved. Then it can be seen that as the EVA works successfully, $x_i(t)$ gradually becomes a source signals $s_{\rho_i}(t-k_i)$. Namely, the diagonal elements of $\tilde{\boldsymbol{\Lambda}}$ in (19) gradually become zeros except for one element corresponding to $s_{\rho_i}(t-k_i)$. This means that when the eigenvectors of $\tilde{\boldsymbol{R}}^\dagger \tilde{\boldsymbol{B}}_i(t)$ are calculated for achieving the BD, it is only enough that we select the eigenvector corresponding to the absolute maximum eigenvalue of $\tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{B}}_i(t)$. This is the reason why we can circumvent the tedious task by using the reference signal. After all, the EVA is implemented as follows:

Set initial values: $\tilde{\boldsymbol{w}}_i(0)$, $\tilde{\boldsymbol{R}}(0)$, $\tilde{\boldsymbol{B}}_i(0)$
for $t_l = 1 : t_{l_{all}}$
  for $t = t_d(t_l - 1)+1{:}t_d t_l$
    $x_i(\mathrm{t}) = \tilde{\boldsymbol{w}}_i^T(t_l - 1)\tilde{\boldsymbol{y}}_i(t)$
    Calculate $\tilde{\boldsymbol{R}}(t)$ and $\tilde{\boldsymbol{B}}_i(t)$ by a moving average.
  end
  Calculate the eigenvector $\tilde{\boldsymbol{w}}_i(t_l)$ associated with the absolute maximum eigenvalue $|\lambda_i|$ from (22).
end

where $t_{l_{all}}$ denotes the total number of iterations and t_d denotes the number of data samples for estimating the matrices $\tilde{\boldsymbol{R}}(t)$ and $\tilde{\boldsymbol{B}}_i(t)$. Note that $\tilde{\boldsymbol{R}}$ is not needed to estimate iteratively, but for the sake of our convenience, this way is adopted.

Here it is worth noting that when the above algorithm is implemented, it may happen that each output of a deconvolver provides the same source signal. Therefore, in order to avoid such a situation, we apply a deflation approach, that is, the Gram-Schmidt decorrelation [1] to the eigenvectors $\tilde{\boldsymbol{w}}_i(t_l)$ for $i = \overline{1, n}$.

4 Simulation Results

To demonstrate the validity of the proposed method, many computer simulations were conducted. Some results are shown in this section. The unknown system $\boldsymbol{H}(z)$ was set to be the same channel with two inputs and three outputs as in [9]. Also, other setup conditions, that is, the source signals $s_i(t)$'s, the noises $n_i(t)$'s,

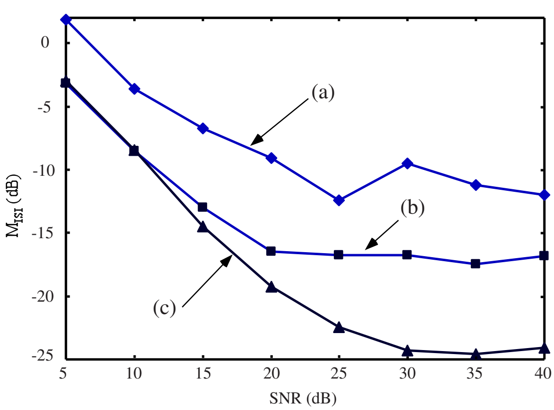

Fig. 2. The performances of the proposed EVA and our conventional EVA with varying SNR levels, in the cases of 5,000 data samples

and their SNR levels were the same as in [9]. As a measure of performances, we used the *multichannel intersymbol interference* (M_{ISI}) [5], which was the average of 30 Monte Carlo runs. In each Monte Carlo run, the number of iterations $t_{l_{all}}$ was set to be 10, and the number of data samples t_d was set to be 5,000. For comparison, our conventional EVA in [9] was used, where the conventional EVA does not need deflation approaches.

Fig. 2 shows the results of performances of the EVAs when the SNR levels were respectively taken to be 5 through 40 dB for every 5 dB, where there are three kinds of reference signals, (a) $x(t) = \sum_{i=1}^{3} f_i(5)y_i(t-5)$, where each parameter $f_i(5)$ was randomly chosen from a Gaussian distribution with zero mean and unit variance, (b) $x(t) = f_2(2)y_2(t-2)$, where $f_2(2)$ also was randomly chosen from the Gaussian distribution, (c) $x_i(t) = \tilde{\boldsymbol{w}}_i^T(t-1)\tilde{\boldsymbol{y}}_i(t)$, $i = \overline{1,3}$. The last reference signal (c) corresponds to the proposed EVA, while the other two (a) and (b) correspond to our conventional EVA.

From Fig. 2, it can be seen that the proposed EVA provides better performances than our conventional EVA [9].

5 Conclusions

We have proposed an EVA for solving the BD problem. Using the output of a deconvolver as a reference signal, the tedious task of our conventional EVA can be circumvented. The simulation results have demonstrated the effectiveness of the proposed EVA. However, from the simulation results, one can see that all our EVAs have such a drawback that it is sensitive to Gaussian noise. Therefore, as a further work, we will propose an EVA having such a property that the BD can be achieved as little insensitive to Gaussian noise as possible.

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