An Adaptive Super-Exponential Deflation Algorithm for Blind Deconvolution of MIMO Systems Using the QR-factorization of Matrix Algebra

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***Abstract*—The multichannel blind deconvolution of finite-impulse response (FIR) or infinite-impulse response (IIR) systems is investigated using the multichannel super-exponential deflation methods. We propose a new adaptive approach to the multichannel super-exponential deflation methods using the QR-factorization of matrix algebra and the higher-order cross correlations of the (channel) system and equalizer outputs. In order to see the effectiveness of the proposed approach, many computer simulations are carried out for time-invariant MIMO systems along with time-variant MIMO systems. It is shown through computer simulations that the proposed approach is effective for time-invariant systems, but is not so effective for time-variant systems as we expected in advance.**

I. Introduction

Multichannel blind deconvolution has recently received attention in such fields as digital communications, image processing and neural information processing [1],[2].

Recently, Shalvi and Weinstein proposed an attractive approach to single-channel blind deconvolution called the *super-exponential method* (SEM) [3]. Extensions of their idea to multichannel deconvolution were presented by Inouye and Tanebe [4], Martone [5], [6], and Yeung and Yau [7]. In particular, Inouye and Tanebe [4] proposed the multichannel super-exponential deflation method (MSEDM) using the second-order correlations. Martone [6], and Kawamoto, Kohno and Inouye [8] proposed MSEDM's using the higher-order correlations for instantaneous mixtures or constant channel systems. Kohno, Inouye and Kawamoto [10] proposed MSEDM using the higher-order correlations for convolutive mixtures or dynamical channel systems. In the meantime, Kohno, Inouye, Kawamoto and Okamoto proposed two type of adaptive multichannel super-exponential algorithms (AMSEA's), the one in covariance (correlation or Kalman-filter) form and the other in QR-factorization form, for the degenerate rank case of the correlations matrices [9].

In the present paper, we propose an adaptive multichannel super-exponential deflation algorithm (AMSEDA) using the QR-factorization and the higher-order correlations for convolutive mixtures or dynamical channel systems, and show the effectiveness of the proposed method by computer simulations. Another AMSEDA using the covariance form will be appeared in a forthcoming paper.

The present paper uses the following notation: Let Z denote the set of all integers. Let $\boldsymbol{C}^{m\times n}$ denote the set of all $m \times n$ matrices with complex components. The superscripts T, $*$, H and $\dagger$ denote, respectively, the transpose, the complex conjugate, the complex conjugate transpose (Hermitian) and the (Moore-Penrose) pseudoinverse operations of a matrix. Let $i = \overline{1,n}$ stand for $i = 1, 2, \cdots, n$.

II. Assumptions and Preliminaries

We consider an MIMO channel system with n inputs and m outputs as described by

$$\boldsymbol{y}(t) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)}\boldsymbol{s}(t-k), \qquad t \in Z, \tag{1}$$

where

$\boldsymbol{s}(t)$ n-column vector of input (or source) signals,
$\boldsymbol{y}(t)$ m-column vector of channel outputs,
$\boldsymbol{H}^{(k)}$ $m \times n$ matrix of impulse responses.

The transfer function of the channel system is defined by

$$\boldsymbol{H}(z) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}^{(k)} z^k, \qquad z \in C. \tag{2}$$

For the time being, it is assumed for theoretical analysis that the noise is absent in (1).

To recover the source signals, we process the output signals by an $n \times m$ equalizer (or deconvolver) $\boldsymbol{W}(z)$ described by

$$\boldsymbol{z}(t) = \sum_{k=-\infty}^{\infty} \boldsymbol{W}^{(k)}\boldsymbol{y}(t-k), \qquad t \in Z. \tag{3}$$

The objective of multichannel blind deconvolution is to construct an equalizer that recovers the original source signals only from the measurements of the corresponding outputs.

We put the following assumptions on the systems and the source signals.

A1) The transfer function $\boldsymbol{H}(z)$ is stable and has full column rank on the unit circle $|z| = 1$ [this implies that the unknown system has less inputs than outputs, i.e., $n \leq m$, and there exists a left stable inverse of the unknown system].

A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a complex, zero-mean, non-Gaussian random vector process with ele-

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ment processes $\{s_i(t)\}$, $i=\overline{1,n}$ being mutually independent. Moreover, each element process $\{s_i(t)\}$ is an i.i.d. process with a nonzero variance σ_i^2 and a nonzero fourth-order cumulant γ_i. The variances σ_i^2's and the fourth-order cumulants γ_i's are unknown.

A3) The equalizer $\boldsymbol{W}(z)$ is an FIR system of sufficient length L so that the truncation effect can be ignored.

Remark 1: As to A1), if the channel system $\boldsymbol{H}(z)$ is FIR, then a condition of the existence of an FIR equalizer is rank $\boldsymbol{H}(z)=n$ for all nonzero $z\in C$ [11]. Moreover, if $\boldsymbol{H}(z)$ is irreducible, then there exists an equalizer $\boldsymbol{W}(z)$ of length $L\leq n(K-1)$, where K is the length of the channel system [11]. Besides, it is shown that there exists generically (or except for pathological cases) an equalizer $\boldsymbol{W}(z)$ of length $L\leq\lceil\frac{n(K-1)}{m-n}\rceil$, where $\lceil x\rceil$ stands for the smallest integer that is greater than equal to x.

Let us consider an FIR equalizer with the transfer function $\boldsymbol{W}(z)$ given by

$$\boldsymbol{W}(z)=\sum_{k=L_1}^{L_2}\boldsymbol{W}^{(k)}z^k, \tag{4}$$

where the length $L:=L_2-L_1+1$ is taken to be sufficiently large. Let $\tilde{\boldsymbol{w}}_i$ be the Lm-column vector consisting of the tap coefficients (corresponding to the ith output) of the equalizer defined by

$$\tilde{\boldsymbol{w}}_i:=\left[\boldsymbol{w}_{i,1}^T,\boldsymbol{w}_{i,2}^T,\cdots,\boldsymbol{w}_{i,m}^T\right]^T\in\ \boldsymbol{C}^{mL}, \tag{5}$$

$$\boldsymbol{w}_{i,j}=\left[w_{i,j}{}^{(L_1)},w_{i,j}{}^{(L_1+1)},\cdots,w_{i,j}{}^{(L_2)}\right]^T\in\ \boldsymbol{C}^{L}, \tag{6}$$

where $w_{i,j}{}^{(k)}$ is the (i,j)th element of matrix $\boldsymbol{W}^{(k)}$.

Inouye and Tanebe [4] proposed the *multichannel super-exponential algorithm* for finding the tap coefficient vectors $\tilde{\boldsymbol{w}}_i$'s of the equalizer $\boldsymbol{W}(z)$, of which each iteration consists of the following two steps:

$$\tilde{\boldsymbol{w}}_i^{[1]}=\tilde{\boldsymbol{R}}_L^{\dagger}\tilde{\boldsymbol{d}}_i \qquad for\ i=\overline{1,n}, \tag{7}$$

$$\tilde{\boldsymbol{w}}_i^{[2]}=\frac{\tilde{\boldsymbol{w}}_i^{[1]}}{\sqrt{\tilde{\boldsymbol{w}}_i^{[1]H}\tilde{\boldsymbol{R}}_L\tilde{\boldsymbol{w}}_i^{[1]}}} \quad for\ i=\overline{1,n}, \tag{8}$$

where $(\cdot)^{[1]}$ and $(\cdot)^{[2]}$ stand respectively for the result of the first step and the result of the second step. Let $\tilde{\boldsymbol{y}}(t)$ be the Lm-column vector consisting of the L consecutive inputs of the equalizer defined by

$$\tilde{\boldsymbol{y}}(t):=\left[\bar{\boldsymbol{y}}_1(t)^T,\bar{\boldsymbol{y}}_2(t)^T,\cdots,\bar{\boldsymbol{y}}_m(t)^T\right]^T\ \in\ \boldsymbol{C}^{mL}, \tag{9}$$

$$\bar{\boldsymbol{y}}_i(t):=\left[y_i(t-L_1),y_i(t-L_1-1),\cdots,y_i(t-L_2)\right]^T\in\boldsymbol{C}^{L}, \tag{10}$$

where $y_i(t)$ is the ith element of the output vector $\boldsymbol{y}(t)$ of the channel system in (1). Then the correlation matrix $\tilde{\boldsymbol{R}}_L$ is represented as

$$\tilde{\boldsymbol{R}}_L=E\left[\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)\right]\in\ \boldsymbol{C}^{mL\times mL}, \tag{11}$$

and the fourth-order cumulant vector $\tilde{\boldsymbol{d}}_i$ is represented as

$$\begin{aligned}\tilde{\boldsymbol{d}}_i\ =\ & E\left[|z_i(t)|^2z_i(t)\tilde{\boldsymbol{y}}^*(t)\right]\\ & -2E\left[|z_i(t)|^2\right]E\left[z_i(t)\tilde{\boldsymbol{y}}^*(t)\right]\\ & -E\left[z_i{}^2(t)\right]E\left[z_i{}^*(t)\tilde{\boldsymbol{y}}^*(t)\right]\ \in\ \boldsymbol{C}^{mL},\end{aligned} \tag{12}$$

where E$[x]$ denotes the expectation of a random variable x. We note that the last term can be ignored in case of E$[s_i{}^2(t)]$=0 for all $i=\overline{1,n}$, in which case E$[z_i{}^2(t)]$=0 for all $i=\overline{1,n}$.

III. The Adaptive Super-Exponential Algorithm Using the QR-factorization

Kohno, Inouye, Kawamoto and Okamoto proposed two types of AMSEA's, the one in covariance (correlation or Kalman-filter) form and the other in QR-factorization form, for the degenerate rank case of the correlations matrices [9]. Except for the case when the number of outputs equals the number of inputs, i.e., $m=n$, the correlation matrix $\tilde{\boldsymbol{R}}_L$ is not of full rank. Situations with the number of independent sources (or inputs) being strictly less than the number of sensors (or outputs) are often encountered in various applications such as digital communication, image processing and neural information processing. Moreover, if the underlying channel system exhibits slow changes in time, processing all the available data jointly is not desirable, even if we can accommodate the computational and storage loads of the batch algorithm in (7) and (8), because different data segments correspond to different channel responses. In such a case, we want to have an adaptive algorithm which is capable of tracking the varying characteristics of the channel system.

Consider the batch algorithm in (7) and (8). The equation (8) constraints the length of vector $\tilde{\boldsymbol{w}}_i$ to equal one, and thus we assume this constraint is always satisfied using a normalization or an automatic gain control (AGC) of $\tilde{\boldsymbol{w}}_i$ at each time t. To develop an adaptive version of (7), we must specify the dependency of each time t and rewrite (7) as

$$\tilde{\boldsymbol{w}}_i(t)=\tilde{\boldsymbol{R}}_L^{\dagger}(t)\tilde{\boldsymbol{d}}_i(t)\ \ ,\ \ i=\overline{1,n}. \tag{13}$$

Here the subscript L of $\tilde{\boldsymbol{R}}_L(t)$ is omitted for simplicity hereafter. The recursions for time-updating of matrix $\tilde{\boldsymbol{R}}(t)$ and vector $\tilde{\boldsymbol{d}}_i(t)$ in (13) are given as

$$\tilde{\boldsymbol{R}}(t)\ =\ \alpha\tilde{\boldsymbol{R}}(t-1)+(1-\alpha)\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t), \tag{14}$$

$$\tilde{\boldsymbol{d}}_i(t)\ =\ \alpha\tilde{\boldsymbol{d}}_i(t-1)+(1-\alpha)\tilde{\boldsymbol{y}}^*(t)\tilde{z}_i(t), \tag{15}$$

where

$$\tilde{z}_i(t):=(|z_i(t)|^2-2<|z_i(t)|^2>)z_i(t)-<z_i^2(t)>z_i^*(t). \tag{16}$$

Here $<|z_i(t)|^2>$ and $<z_i^2(t)>$ denote respectively the estimates of $E\left[|z_i(t)|^2\right]$ and $E\left[z_i(t)^2\right]$ at time t, α is a positive constant close to, but less than one, which accounts for some exponential weighting factor or forgetting factor [13].

The AMSEA using the QR-factorization is introduced on the basis of the following lemma, the so-called QR-factorization of a general matrix $\boldsymbol{A}$.

Lemma 1 [12],[14]**:** Given an $n\times n$ Hermitian $\boldsymbol{A}\in\boldsymbol{C}^{n\times n}$. Let r be a chosen integer satisfying $|\lambda_r|>|\lambda_{r+1}|$, where the eigenvalues $\lambda_1,\lambda_2,\cdots,\ \lambda_n$ of $\boldsymbol{A}$ are arranged in decreasing order of magnitude. Given an $n\times r$ matrix $\boldsymbol{Q_0}$ with orthonormal columns and generate a sequence of matrices $\{\boldsymbol{Q}_k\}\subset\boldsymbol{C}^{n\times r}$ as follows:

$$\boldsymbol{Z}_k=\boldsymbol{A}\boldsymbol{Q}_{k-1}, \tag{17}$$

$$\boldsymbol{Q}_k\boldsymbol{R}_k = \boldsymbol{Z}_k \quad : \textit{QR-factorization}, \tag{18}$$

where $\boldsymbol{Q}_k \in \boldsymbol{C}^{n\times r}$ is a matrix with orthonormal columns and $\boldsymbol{R}_k \in \boldsymbol{C}^{r\times r}$ is an upper triangular matrix. If $\boldsymbol{Q_0}$ is not unfortunately chosen, then the sequence $\{\boldsymbol{Q}_k\}$ converges to a matrix of r dominant eigenvectors, and the upper triangular sequence $\{\boldsymbol{R}_k\}$ converges the diagonal matrix of r dominant eigenvalues.

By applying Lemma 1 for calculating the pseudoinverse of $\tilde{\boldsymbol{R}}(t)$, we have the following theorem which gives an adaptive solution $\tilde{\boldsymbol{w}}_i(t)$ of (13) from $\boldsymbol{Q}_r(t-1)$, $\boldsymbol{Q}_r(t-2)$, $\tilde{\boldsymbol{d}}_i(t-1)$, $\tilde{\boldsymbol{y}}(t)$ and $z_i(t)$ (where, for example, $\boldsymbol{Q}_r(t-1) \in \boldsymbol{C}^{mL\times r}$ represents approximately r dominants eigenvectors of $mL \times mL$ matrix $\tilde{\boldsymbol{R}}(t-1)$).

Theorem 1: Let r be fixed as $r = nL$, where n is the number of the inputs of the channel system in (1) and L is the length of the equalizer in (4). Then an adaptive solution $\tilde{\boldsymbol{w}}_i(t)$ of (13) is

$$\tilde{\boldsymbol{w}}_i(t) = \boldsymbol{Q}_r(t-1)\boldsymbol{R}_r^{-1}(t)\boldsymbol{Q}_r^H(t)\tilde{\boldsymbol{d}}_i(t), \tag{19}$$

where $\boldsymbol{Q}_r(t)$ and $\boldsymbol{R}_r(t)$ are obtained by the QR decomposition of matrix $\boldsymbol{Z}(t)$ defined by $\boldsymbol{Z}(t) := \tilde{\boldsymbol{R}}(t)\,\boldsymbol{Q}_r(t-1)$, which is decomposed as

$$\begin{aligned}\boldsymbol{Z}(t) &= \boldsymbol{Q}_r(t)\boldsymbol{R}_r(t) \in \boldsymbol{C}^{mL\times r},\\ &\boldsymbol{Q}_r(t) \in \boldsymbol{C}^{mL\times r},\quad \boldsymbol{R}_r(t) \in \boldsymbol{C}^{r\times r},\end{aligned} \tag{20}$$

and the update of $\boldsymbol{Z}(t)$ is

$$\begin{aligned}\boldsymbol{Z}(t) &= \alpha_1\boldsymbol{Z}(t-1)\boldsymbol{Q}_r^H(t-2)\boldsymbol{Q}_r(t-1)\\ &\quad + (1-\alpha_1)\tilde{\boldsymbol{y}}^*(t)\tilde{\boldsymbol{y}}^T(t)\boldsymbol{Q}_r(t-1).\end{aligned} \tag{21}$$

The update of $\tilde{\boldsymbol{d}}_i(t)$ is

$$\tilde{\boldsymbol{d}}_i(t) = \alpha_2\tilde{\boldsymbol{d}}_i(t-1) + (1-\alpha_2)\tilde{\boldsymbol{y}}^*(t)\tilde{z}_i(t), \tag{22}$$

where

$$\tilde{z}_i(t) := (|z_i(t)|^2 - 2 < |z_i(t)|^2 >)z_i(t) - < z_i^2(t) > z_i^*(t), \tag{23}$$

$$< |z_i(t)|^2 > = \beta < |z_i(t-1)|^2 > + (1-\beta)|z_i(t)|^2, \tag{24}$$

$$< z_i^2(t) > = \beta < z_i^2(t-1) > + (1-\beta)z_i^2(t). \tag{25}$$

Here α_1 and α_2 are positive constants close to, but less than one, which accounts for some exponential weighting factor or forgetting factor [13]. The β is a positive constant less than α_1 and α_2. These equations are initialized by their values appropriately selected or calculated by the batch algorithm in (7) and (8) at an initial time t_0 and used for $t = t_0 + 1, t_0 + 2, \cdots$.

IV. The Adaptive Super-Exponential Deflation Algorithm Using the QR-factorization

The MSEDM proposed by Inouye and Tanebe [4] uses the second-order correlations to estimate the contributions of an extracted source signal to the channel outputs. Kohno, Inouye and Kawamoto [10] proposed MSEDM using the higher-order correlations instead of the second-order correlations to reduce the computational complexity in terms of multiplications and to accelerate the performance of equalization. For the details of the MSEDM using the higher-order correlations, see the equations from (13) through (30) in [10]. In the present paper, we proposed a new AMSEDA which combines the MSEDM using the higher-order correlations with the adaptive algorithm using the QR-factorization form described in the previous chapter.

In our new AMSEDA, the following procedures are carried out in each time when channel outputs are observed.

Before the following procedures are carried out, it is necessary that $\tilde{\boldsymbol{R}}$, $\boldsymbol{Q_r}$, $\tilde{\boldsymbol{d}}_i$ and $\tilde{\boldsymbol{w}}_i$ are initialized.

At first, set $t = t_0$, and set $l = 1$ where l denotes the number of channels (or the sources) equalized.

Then, $\boldsymbol{Z}(t)$ is calculated by (21), $\tilde{\boldsymbol{d}}_1(t)$ is calculated by using from (22) to (25), and $\tilde{\boldsymbol{w}}_1(t)$ is calculated by the two steps (19) and (8). By these procedures, the first equalized output $z_1(t)$ is obtained.

Next, the MSEDM using the higher-order correlations is carried out. We calculate the contribution signals by using the equalized output $z_1(t)$, and remove the contribution signals from the channel outputs in order to define the outputs of a multichannel system with $n-1$ inputs and m outputs. The number of inputs becomes deflated by one. The procedures mentioned above are continued until $l = n$, where we obtain the last equalized output $z_n(t)$ for $t = t_0$. If $t < t_f$ (where t_f is a final time), then set $t = t_0 + 1$ and iterate the same procedures as the previous time t. If $t = t_f$, then stop here. The n equalized outputs $z_1(t), \cdots, z_n(t)$ are obtained for $t = t_0, t_0+1, \cdots, t_f$.

Therefore, the proposed algorithm is summarized as shown in Table 1.

Table 1. The proposed algorithm.

Step	Contents
1	Set $t = t_0$ (where t_0 is an initial time).
2	Set $l = 1$ (where l denotes the number of the channels equalized).
3	Calculate the QR-factorization of $\boldsymbol{Z}(t)$ using (21).
4	Calculate $\tilde{\boldsymbol{d}}_l(t)$ using from (22) through (25).
5	Calculate $\tilde{\boldsymbol{w}}_l(t)$ using (19) and (8).
6	Carry out the deflationary process using the MSEDM with the higher-order correlations [10].
7	If the subscript l is less than n, then set $l = l+1$, and the procedures (from Step 3 through Step 6) are continued until $l = n$.
8	If $t < t_f$ (where t_f is a final time), then set $t = t + 1$ and iterate the procedures from Step 2 through Step 7. If $t = t_f$, then stop here.

V. Simulations

To demonstrate the effectiveness of proposed method, some computer simulations were conducted. We considered an MIMO channel system with two inputs and three outputs, and assumed that the length of channel is three ($K = 3$), that is $\boldsymbol{H}^{(k)}$ in (1) was set to be

$$\boldsymbol{H}(z) = \begin{bmatrix} 1.00 + 0.15z + 0.10z^2 & 0.65 + 0.25z + 0.15z^2 \\ 0.50 - 0.10z + 0.20z^2 & 1.00 + 0.25z + 0.10z^2 \\ 0.60 + 0.10z + 0.40z^2 & 0.10 + 0.20z + 0.10z^2 \end{bmatrix} \tag{26}$$

The length of equalizer was chosen to be seven (L=7). We set the values of the tap coefficients to be zero expect for $w_{12}(4) = w_{21}(4) = 1$. Two source signals were 4-PSK and 8-PSK signals, respectively. For recovering first source signal, the initial values of $\tilde{\boldsymbol{R}}$ and $\tilde{\boldsymbol{d}}_i$

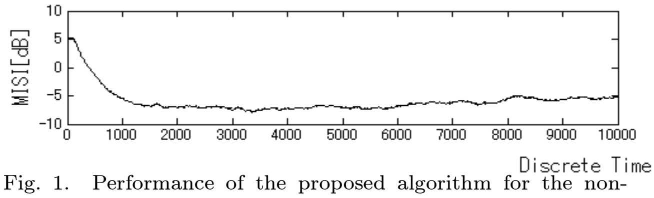

Fig. 1. Performance of the proposed algorithm for the non-adaptive model.

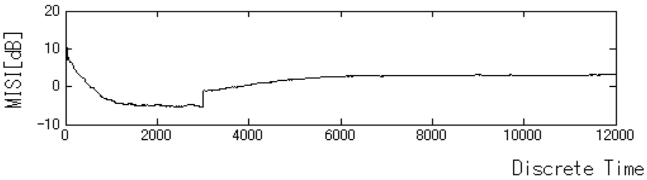

Fig. 2. Performance of the proposed algorithm for the adaptive model.

were estimated using 5,000 data samples. For recovering second source signal, the initial value of $\tilde{\boldsymbol{R}}$ was set the identity matrix $\boldsymbol{I}$ and $\boldsymbol{Q_r}(0)$ was set the matrix $[\boldsymbol{I}, \boldsymbol{0}]^T$. The values of α_1, α_2 and β were chosen as $\alpha_1 = 0.999$, $\alpha_2 = 0.99999$ and $\beta = 0.05$, respectively. Besides, we used the fourth-order correlation method for subtracting the contributions of an extracted source signal to the channel outputs. As a measure of performance, we use the multichannel intersymbol interference ($\mathrm{M_{ISI}}$) defined in the logarithmic (dB) scale by

$$\mathrm{M_{ISI}} := 10 \log_{10} \left[\sum_{i=1}^{n} \frac{|\Sigma_{j=1}^{n} \Sigma_{t=-\infty}^{\infty} |g_{ij}(t)|^2 - |g_{i\cdot}|^2_{\max}|}{|g_{i\cdot}|^2_{\max}} + \sum_{j=1}^{n} \frac{|\Sigma_{i=1}^{n} \Sigma_{t=-\infty}^{\infty} |g_{ij}(t)|^2 - |g_{\cdot j}|^2_{\max}|}{|g_{\cdot j}|^2_{\max}} \right], \quad (27)$$

where $|g_{i\cdot}|^2_{\max}$ and $|g_{\cdot j}|^2_{\max}$are respectively defined by

$$|g_{i\cdot}(\cdot)|^2_{\max} := \mathrm{Max}_{j=1,\cdots,n} \mathrm{Max}_{-\infty<t<\infty} |g_{ij}(t)|^2 \quad (28)$$

$$|g_{\cdot j}(\cdot)|^2_{\max} := \mathrm{Max}_{i=1,\cdots,n} \mathrm{Max}_{-\infty<t<\infty} |g_{ij}(t)|^2 \quad (29)$$

Fig. 1 shows $\mathrm{M_{ISI}}$ of the performance results for the time-invariant channel system obtained by using 10,000 data samples. It can be seen from Fig. 1 that our proposed method quickly deconvolved all source signals and it is effective for the time-invariant channel system. Fig. 2 shows $\mathrm{M_{ISI}}$ of the performance results for the time-variant channel system obtained by using 12,000 data samples. The last matrix $\boldsymbol{H}^{(2)}$ of the impulse response of the channel was varied by adding 0.3 to all its elements at discrete time t=3,000. It can be seen from Fig. 2 that our proposed method is not so effective as we expected earlier for the time-variant system.

VI. Conclusions

We have considered the problem of adaptive multichannel blind deconvolution based on the super-exponential algorithms using deflation methods proposed by Inouye and Tanebe [4]. In this paper, we proposed a new approach to the adaptive multichannel deflationary blind deconvolution using the QR-factorization and the higher-order correlations. In order to see the effectiveness of the proposed approach, we have considered computer simulations for two types of MIMO systems, that is, the first one is time-invariant and the second one is time-variant. It has been shown through computer simulations that the proposed approach is effective for time-invariant systems, but is not so effective for time-variant systems as we expected beforehand. One of reasons why it is not so effective for time-variant systems is that the values of $\tilde{\boldsymbol{d}}_i(t)$ in (22) change after the channel characteristic varies. These changes make the proposed algorithm unstable. In order to suppress these changes to be small, it is necessary to choose appropriately the value of α_2 in (22) or to introduce additional terms $\tilde{\boldsymbol{d}}_i(t-k)$'s (where $k = \overline{2, M}$) in the right-hand side of (22). To circumvent this issue, another AMSEDA using the covariance form will be developed in a forthcoming paper.

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